ABSTRACT

Given that the homogeneous functions of the degree r, with $r \in \Re$, form a real vector space, there must exist a base for this vector space. The objective of this paper is to find this base.

In this paper it is shown that the homogeneous polynomials of the form $\{x^\alpha y^{r-\alpha}\}_{\alpha\in[a,b]}$ constitute a base for the vector space of the degree r. Using Hilbert's theory of the compact operators in space, it shows that the set $\{x^\alpha y^{r-\alpha}\}_{\alpha\in[a,b]}$ is linearly independent and using the Hahn-Banach theorem it shows that any homogenous function can be written as a linear combination of them. That is, $f(x,y) = \int_a^b h(\alpha)x^\alpha y^{r-\alpha}d\alpha$ with $f(x,y)$ a homogenous function of $\Re^2$ and $(x,y) \in Dom(f)$.

The homogeneous polynomials of the form $x^\alpha y^{r-\alpha}$ are known in economy as the Cobb-Douglas production functions, with the variable x being the one that measures the units of invested capital and the variable y the one that measures the units of work assigned to this Cobb-Douglas production function. In this sense, the previous theoretical result can have practical applications in the field of Economics.

Key words: Homogeneous functions, Hilbert spaces, Banach spaces, Cobb-Douglas functions.