10. Functional Analysis

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Solution of Nonlinear Operator Equations by the Oseen Method

Iterative methods are a universal tool for the investigation and solution of operator equations. We consider the nonlinear operator equation in the form

$$u = N\,(u) \quad , \qquad u \in K \ . \tag{1}$$

Here E is a Banach space, K is a subset of E, and $N : K \to E$. The solution of this equation is called the fixed point of the operator N. The problem of solving the nonlinear equation

$$F\,(u) = 0 \quad , \qquad u \in K \tag{2}$$

for some operator $F : K \subseteq E \to E$ can be reduced to an equivalent fixed-point problem of the form (1) by setting $N(v) = v - F(v)$, or more generally, $N(v) = v - c_0 F(v)$ for some constant scalar $c_0 \neq 0$. Thus any result on the problem (2) can be translated into a result for the equation (1) and vice versa.
One can study the general iterative process for nonlinear operator equations, when two or more approaches are introduced into the equation (2)

$$F_k\,(u_{k+1}, u_k, u_{k-1}, ...) = 0 \quad . \tag{3}$$

This process will be called the multi-step implicit iterative process.
We have the simple one-step case, when only two approaches are in equation (2)

$$F_k\,(u_{k+1}, u_k) = 0 \quad . \tag{4}$$

We introduce the designations

$$u_{k+1} = v \quad , \quad u_k = w \quad , \quad F_k = M$$

and rewrite the equation (4) in the form

$$M\,(v\,, w) = 0 \quad . \tag{5}$$

The operator M can be linear or nonlinear in regard to its first argument v.
The case, when the operator M is linear with respect to a variable v , is called the Oseen linearization method [1] . This linearization was launched in 1910 Oseen, who applied it to the solution of the Navier-Stokes equations for the slow flows, but Oseen did not connect his linearization scheme with a method of successive approximations.
We use the general implicit function theory for the analysis of this method. Applying the Oseen method we have obtained the solutions of various nonlinear problems from continuum mechanics. The numerical results are in congruent with the results, that were obtained by other numerical methods.
The application of the Oseen method for the solution of some inverse problems is investigated too. As an example of elliptic inverse problems is considered the inverse conductivity problem also

called electrical impedance tomography (EIT) [2].
At last we use the Oseen method for the solution of a nonlinear ill-posed operator equation

$$F(u) = y \quad , \tag{6}$$

where $F : X \rightarrow Y$ is a map between two Hilbert spaces X and Y. If only noisy data y^δ with

$$||y^\delta - y|| \leq \delta \tag{7}$$

are available, then the problem of solving (6) has to be regularized.
We would like to believe that the report attracts the attention of the mathematicians and engineers to the Oseen method and to the utilization of this method for the solution of some nonlinear well-posed and ill-posed problems.

References

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