Section number 10

ON THE CATEGORY OF HIGHEST WEIGHT REPRESENTATIONS OF LIE GROUP $D_{n-1/2}$

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Let us denote by T_m a finite–dimensional irreducible representation of a complex simple Lie group G in one of the series A_n, B_n, C_n or D_n, m is the heighest weight of T_m. Let G_0 be a maximal subgroup of the smaller rank of the same name (A_{n-1}, B_{n-1}, C_{n-1} or D_{n-1}, respectively). Restriction $T_m|_{G_0}$ is quite reducible and the corresponding branching rule can be written in the uniform way for all series:

$$T_m|_{G_0} = \bigoplus_{l \to m} \left(\bigoplus_{t \to l} T_t \right),$$

where $l \to m$, $t \to l$ are some subordination conditions for highest weights expressed in terms of inequalities system. This branching rule suggests an idea on possible hidden symmetry. And indeed, as it was shown in author's works, the space $\bigoplus\limits_{t \to l} T_t$ has a structure not only of G_0–module but also a module structure over some intermediate between G_0 and G Lie group. Let us denote by $D_{n-1/2}$ the intermediate Lie group corresponding to the series D. It can be defined also as the stabilizer of a non–zero isotropic vector in space of the standard representation of D_n.

The chain of embeddings

$$D_1 \subset D_{2-1/2} \subset D_2 \subset \cdots \subset D_{n-1/2} \subset D_n \tag{1}$$

differs from spectral analysis point of view from the classic one

$$D_1 \subset B_1 \subset D_2 \subset \cdots \subset B_{n-1} \subset D_n. \tag{2}$$

Unlike (2), all enbeddings in (1) are regular and it make possible to construct canonical weight basis in representation space of T_m. Remember that Gel'fand–Tsetlin basis, responding to (2), is not weight.

One can make the spectrum of D_{n-1}–module T_m multiplicity free by constructing a $D_{n-1/2}$–module filtration in T_m. If we shall consider the set of all possible finite–dimensional irreducible representations T_m of Lie group D_n, then the set of corresponding filtration factors forms the category $\mathcal{L}$ of finite–dimensional cyclic $D_{n-1/2}$–modules with highest weight. We have obtained the next description of this category: $\mathcal{L} = \{\mathcal{L}(\lambda_1, \ldots, \lambda_{n-1}; m_{n-1}, m_n)\}$, where components of highest weight satisfy the inequalities system:

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{n-2} \geq m_{n-1} \geq \lambda_{n-1} \geq -m_n \geq -m_{n-1}.$$

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