Dilations and Multi-Dimensional Moment Problems

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ABSTRACT

Given a family $\{A_{\mathbf{n}}\}_{\mathbf{n}\in\mathbf{Z}_{+}^{\Omega}}$ of operators between a Hilbert spaces $\mathcal{H}$ and $\mathcal{K}$ we characterize the existence of a multi-contraction $\mathbf{T} = \{T_{\omega}\}_{w\in\Omega}$ on $\mathcal{K}$ such that $A_{\mathbf{n}} = \mathbf{T^n}A_{\mathbf{0}}$, $\mathbf{n} \in \mathbf{Z}_{+}^{\Omega}$.

More precisely the following conditions are equivalent:

(a) *The problem above has a solution having regular unitary dilation;*

(b)

$$\| \sum_{\mathbf{m},\mathbf{m'}} A_{\mathbf{m}+\mathbf{m'}}(h_{\mathbf{m},\mathbf{m'}})\|^2 \leq \sum_{\mathbf{m},\mathbf{m'},\mathbf{n},\mathbf{n'}} \langle A_{(\mathbf{m}-\mathbf{n})^{+}+\mathbf{m'}}(h_{\mathbf{m},\mathbf{m'}}), A_{(\mathbf{m}-\mathbf{n})^{-}+\mathbf{n'}}(h_{\mathbf{n},\mathbf{n'}})\rangle,$$

for every finite family $\{h_{\mathbf{n},\mathbf{n'}}\}_{\mathbf{n},\mathbf{n'}\in\mathbf{Z}_{+}^{\Omega}}$ of vectors in $\mathcal{H}$;

(c)

$$\sum_{\mathbf{m},\mathbf{m'},\mathbf{n},\mathbf{n'}} \langle A_{(\mathbf{m}-\mathbf{n})^{+}+\mathbf{m'}}(h_{\mathbf{m},\mathbf{m'}}), A_{(\mathbf{m}-\mathbf{n})^{-}+\mathbf{n'}}(h_{\mathbf{n},\mathbf{n'}})\rangle \geq 0, \quad \{h_{\mathbf{n},\mathbf{n'}}\}_{\mathbf{n},\mathbf{n'}\in\mathbf{Z}_{+}^{\Omega}} \subset \mathcal{H} \text{ finite};$$

(d)

$$\sum_{v\subset u}(-1)^{|v|}\| \sum_{\mathbf{n}} A_{\mathbf{n}+e(v)}(h_{\mathbf{n}})\|^2 \geq 0, \quad u \subset \Omega \text{ and } \{h_{\mathbf{n}}\}_{\mathbf{n}\in\mathbf{Z}_{+}^{\Omega}} \subset \mathcal{H} \text{ finite}.$$

The theorem is a multi-dimensional analogue for some well-known operator moment problems due to Sebestyén (case $|\Omega| = 1$) and extends some recent results due to Găvruţă and Păunescu [GP] (case $|\Omega| = 2$) and to the authors [PS] (general case).

Other similar moment problems concerning subnormality, applications to families of commuting orthogonal projections and further developments are also considered.

[GP] P.Găvruţă and D.Păunescu, Sebestyén's moment problem and regular dilations, *Acta Math. Hung.* **94**(2002), 223-232.

[PS] D. Popovici and Z. Sebestyén, Sebestyén moment problem: The multi-dimensional case, *Proc. A.M.S.*, 2004 (to appear, published electronically 01.12.2003).

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