Recently, Robin's function and Robin's capacity, generalizations of Green's function and logarithmic capacity respectively, were studied intensively in connection with potential-theoretic problems as well as with physical problems, in particular, their interpretations in terms of minimum energy. Our goal is to give another physical interpretation that comes from free boundary problems for a viscous flow in Hele-Shaw cells with an obstacle inside. We will connect the rate of area change of the phase domain with the rate of change of Robin's reduced modulus of the free boundary in order to get an isoperimetric inequality.