The Nondegenerated Hausdorff Matrix Moment Problem

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This work deals with the nondegenerated matrix version of the moment problem (MMP) on an interval of $\mathbb{R}$. For its treatment, the method of Fundamental Matrix Inequalities created by V.P. Potapov [1] is basic. Through it, an explicit solution of the MMP problem, also known as the Hausdorff Matrix Moment Problem, is given. The statement of the MMP problem is the following:

MMP:
Let $(s_j)_{j=0}^k$ be a sequence of $m \times m$ Hermitian matrices. Find the set $\mathcal{M}_{\geq}^{m\times m}([a,b], \mathfrak{B} \cap [a,b]; (s_j)_{j=0}^k)$ of all nonnegative Hermitian measures σ from $\mathcal{M}_{\geq}^{m\times m}([a,b], \mathfrak{B} \cap [a,b])$, such that the condition

$$\int\limits_{[a,b]} t^j \sigma(dt) = s_j$$

for all $j \in \{0, \cdots, k\}$ holds.

By virtue of the Perron–Stieltjes inversion formula, the MMP problem is reduced to a certain functional analytical problem for holomorphic matrix functions s defined in $\mathbb{C} \setminus [a,b]$.

An explicit solution of the problem MMP is given in terms of a linear fractional transformation

$$s := (U_{11}\, p + U_{12}\, q) \cdot (U_{21}\, p + U_{22}\, q)^{-1},$$

where

$$U = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix},$$

called the resolvent matrix of MMP problem, is a $2m \times 2m$ matrix polynomial constructed by the given data $(s_j)_{j=0}^k$, and the column pair $col(p(z), q(z))$ is a Stieltjes pair [2], [3].

References

[1] Kovališina, I.V.; Potapov, V.P.: *An indefinite metric in the Nevanlinna–Pick problem* (in Russian), Dokl. Akad. Nauk Armyan. SSR **59** (1974), 17–22. Engl. transl. in: Collected Papers of V.P. Potapov. Private Translation by T.Ando, Japan 1982, pp. 67–99.

[2] Dyukarev, Yu. M.; Choque Rivero A.E.: *The power moment problem on a compact interval.*(Russian) Mat. Zametki 69 (2001), no.2, 200–213; translation in Math. Notes 69 (2001), no.1-2, 175–187 (Reviewer:Vladimir Bolotnikov) 44.

[3] Choque Rivero A.E.: *PhD Thesis: "A finite Matrix Moment Problem on an interval"* (in German), June 2002, Universität Leipzig. Supervisors: Prof. Dr. Bernd Kirstein (Universität Leipzig), and Doz. Dr. Yury Mihailovich (Kharkov University). This PhD thesis is available at www.geocities.com/achoque/a_b.pdf

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