Section 12. Probability and Statistics

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ON MEAN SQUARE STABILITY OF LINEAR DYNAMICAL SYSTEMS WITH MARKOV DIFFUSION COEFFICIENTS

The paper deals with the linear differential equation in $\mathbb{R}^n$

$$\frac{dx}{dt} = (A + y(t)\,B)\,x(t), \tag{1}$$

where $y(t)$ is a diffusion Markov process in $\mathbf{R}$ defined by a stochastic differential Ito equation of Ornstein-Ulenbeck type. To analyze the trivial solution stability we will use the continuous semigroup of linear operators $\{T(t), t \geq 0\}$ defined by equality

$$(T(t)q)(y) := \mathbf{E}_y^{(s)}\{X^T(t+s,s,y)q(y(t+s))X(t+s,s,y)\}, \tag{2}$$

where $\{X(t+s,s,y), \quad s \geq 0,\ t \geq 0\}$ is the Cauchy matrix-family of (1) under condition $y(s) = y$ and $q(y)$ is bounded continuous $n \times n$ matrix. The infinitesimal operator of this family may be represented as the sum of two closed operators and we can use methods and results of the perturbation operator theory from [1]. Proposed in our paper method, algorithm and example are based on the results published by authors in the papers [2,3]. In first Section a relative boundedness of the infinitesimal operator of semigroup (2) will be proven. Section two reviews some spectral properties of the sum of the commutative operators, which will be used for stability analysis algorithm construction. In Section three it is proven that the exponential mean square stability problem for equation (1) can be formulated as a spectral projector Laurent series decomposition problem for the infinitesimal operator of the semigroup (2). In Section four an algorithm for the above spectral projector decomposition is derived. Section five presents an example involving stochastic stability analysis of a beam under longitudinal small perturbations by Gaussian Markov process with rational spectral density.

References

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