Two stochastic problems for some correlation models of homogeneous and isotropic random fields

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Isotropic random fields take a very important place in simulation of a spatial phenomena in natural sciences and engineering. Isotropic correlation models are in frequent use in different fields of science, among them are geodesy, geology, geomorphology, hydrology, meteorology, agronomy and geophysics. For example, Buell list of correlation functions became standard source of correlation models in meteorology. And very popular exponential correlation is Whittle-Matern model of index 1/2.

We consider homogeneous and isotropic random fields with correlation functions from the ***Buell list*** (e^{-r^2}, $(1+r^2)^{-\nu}$, $(1+r+\kappa r^2)e^{-r}$, $(1+r)e^{-r}$, $(1+r+r^2/3)e^{-r}$, $(1+r-r^2/2)e^{-r}$, $\frac{ae^{-br}-be^{-ar}}{a-b}$, $\frac{\sin r}{r}$, $\frac{2^{1/3}}{\Gamma(2/3)}r^{2/3}K_{2/3}(r)$) and the ***Whittle-Matern class*** ($\frac{2^{1-\nu}}{\Gamma(\nu)}(r)^{\nu}K_{\nu}(r), \nu > 0$).We focus on two stochastic problems for those random fields.

The first problem is on linear extrapolation at the center of a sphere. The homogeneous and isotropic random field $\xi(x)$ is observed on a sphere $S(r)$ of radius r. We obtain explicit formulas for the linear estimator $\widehat{\xi(0)}$ of the random field $\xi(0)$ and for the mean square error of that extrapolation.

The second problem is on estimation of an unknown mean. We assume that random field $\xi(x) = a + \eta(x)$ is observed on a sphere of radius r, where $\eta(x)$ is a homogeneous and isotropic zero mean random field. We consider the estimator $\widehat{a} = \frac{1}{\omega_n}\int\limits_{\mathrm{S_n}}\xi(x)dm_n$ which has the minimal variance among all linear estimators of the parameter a and obtain explicit formulas for the variance of this estimator.