Section 12. Probability and Statistics

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ON EQUILIBRIUM STABILITY OF QUASILINEAR STOCHASTIC FUNCTIONAL DIFFERENTIAL EQUATIONS

The paper is devoted to the Second Lyapunov method development for stochastic functional differential equations. As it has been shown in many papers this method is one of the most power tools for asymptotic Lyapunov stability analysis of dynamical systems with delay (see, for example, [2], and references there). It should be mentioned that success of the Lyapunov approach in many respects depends on happy choice of Lyapunov-Krasovsky type functional. Guided by the ideas and results of [2] and [3] we have succeeded in finding of convenient algorithm for the above functionals construction in a case of stochastic quasilinear functional differential equations. Our approach is based on extension of the resolving semigroup, defined by linear deterministic part [2], to linear operator semigroup, acting in the partially ordered space of countable additive symmetric matrix-valued measures. The weak infinitesimal operator of this semigroup helps to find such a quadratic functional that gives the necessary and sufficient asymptotic stability condition for the equation defined by selected deterministic linear part of analyzing quasilinear equation. And what is more: substituting the solution of the analyzed stochastic equation as an argument of this functional we have got a stochastic process admissive of stochastic differential. This property permits to derive an analogue of Ito formula for the above mentioned stochastic process and, applying the martingale technique in the same way as it has been done in [3], to discuss equilibrium asymptotic stability conditions for initial stochastic nonlinear functional differential equation. It should be mentioned that the proposal approach is very convenient for mean square stability analysis of linear stochastic functional differential equation. As an example we have deduced necessary and sufficient stability condition for delayed stochastic exponent given by Ito type scalar equation $dx(t) = ax(t)+bx(t-\tau)+\sigma x(t)dw(t)$. Some of the mentioned above results have been published in [1].

References

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