Coefficients and expansions in Bessel systems

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We examine general Bessel systems $\{\varphi_y\}_{y\in Y}$ in L^2 on arbitrary measure space X. Functions of the system are indexed by elements of some measure space Y. We pay special attention to particular cases: frames, Riesz bases, orthogonal and orthosimilar systems, wavelet systems, integral transforms of Fourier type.

It's shown that there exist almost everywhere convergent subsequence of expansion in general Bessel system. This subsequence depends only on system not on expansion's coefficients. In particular, we obtain results for frames, continuous wavelet transform and Gabor transform (windowed Fourier transform).

Suppose φ_y are bounded functions. In this case, we have established inequalities that relate the L^p norm, $1 < p < 2$, of a function f to integrals (sums) of certain powers of its "Fourier coefficients" $\hat{f}_y = (f, \varphi_y)$ with weights depending only on $\|\varphi_y\|_\infty$. If $p > 2$ and the corresponding integral (sum) is finite, then $f \in L^p$ and we can reconstruct it in L^p sense as $\int_Y \hat{f}_y \varphi_y \, d\nu(y)$ ($\sum_y \hat{f}_y \varphi_y$). These results generalize theorems of Hausdorff – Young – Riesz, Hardy – Littlewood – Paley et al., which are well-known in the theory of orthogonal and trigonometric series. Specific results for frames, wavelet and Gabor transforms are obtained as well as improvements of classical estimates for orthogonal series.