STABILITY OF CLASSES OF MAPPINGS, QUASICONVEX FUNCTIONS, AND NULL LAGRANGIANS

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Let $n, m, k \in \mathbb{N}$, $2 \le k \le \min\{n, m\}$. Consider continuous functions $F\colon \mathbb{R}^{m\times n} \to \mathbb{R}$ and $G\colon \mathbb{R}^{m\times n} \to \mathbb{R}$ satisfying the following conditions:

(H1) F is quasiconvex (in the sense of C. B. Morrey);
(H2) G is a null Lagrangian (a quasiaffine function);
(H3) F and G are positively homogeneous of order k;
(H4) $\sup\{K \ge 0 : F(\zeta) \ge KG(\zeta),\ \zeta \in \mathbb{R}^{m\times n}\} = 1$;
(H5) $c_F = \inf_{\zeta\in\mathbb{R}^{m\times n},\,|\zeta|=1} F(\zeta) > 0$;
(H6) $\sup_{I=(1\le i_1<\cdots<i_k\le n)} \sum_{J=(1\le j_1<\cdots<j_k\le m)} |\gamma_{JI}| < c_F/\binom{n-1}{k}$ (in the case $k < n$).

Here the coefficients $\gamma_{JI} \in \mathbb{R}$ are from the representation

$$G(\zeta) = \sum_{\substack{J=(1\le j_1<\cdots<j_k\le m),\\ I=(1\le i_1<\cdots<i_k\le n)}} \gamma_{JI} \det{}_{JI}\, \zeta, \quad \zeta \in \mathbb{R}^{m\times n},$$

where $\det_{JI}\zeta$ is the $k\times k$ minor $\det(\zeta_{j_\mu i_\nu})_{\mu,\nu=1,\dots,k}$ of the matrix $\zeta = (\zeta_{ji})_{j=1,\dots,m,\, i=1,\dots,n}$.

Denote by $\mathfrak{G}(K)$, $K \ge 1$, the class of solutions $v \in W^{1,k}_{\mathrm{loc}}(V;\mathbb{R}^m)$, V domains in $\mathbb{R}^n$, to the partial differential relation $F(v'(x)) \le KG(v'(x))$ a.e.

The following stability theorem holds for the class $\mathfrak{G}(1)$.

Theorem. *Let V be a domain in $\mathbb{R}^n$ and let $U \subset V$ be a compact subset in V. Then there exists a function $\alpha = \alpha_{F,G,V,U} : [1, K_0) \to [0, +\infty)$, $\lim_{K\to 1} \alpha(K) = \alpha(1) = 0$, such that for every mapping $v : V \to \mathbb{R}^m$ in the class $\mathfrak{G}(K)$, $1 \le K < K_0$, there is a mapping $u : V \to \mathbb{R}^m$ in the class $\mathfrak{G}(1)$ such that $\|v - u\|_{C(U;\mathbb{R}^m)} \le \alpha(K)\operatorname{diam} v(V)$.*

The special case when F is convex was considered in [1].

References

[1] Egorov, A. A. Stability of Classes of Solutions to Partial Differential Relations Constructed by Convex and Quasiaffine Functions. (Russian). Vodop'yanov, S.K. (ed.), Proceedings on geometry and analysis. International conference-school dedicated to the memory of A. D. Alexandrov (1912–1999), Novosibirsk, Russia, September 9–20, 2002. Novosibirsk: Izdatel'stvo Instituta Matematiki. 275–288 (2003).

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