Section number:13

On a solution of some smoothing problems

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AMS SC: 41A15, 65D10.
Key words: smoothing problem, natural spline of one variable.

We consider the problem of smoothing of an unknown function g from the Sobolev space $H^q[a,b]$ on the basis of the discrete information given by the values of n linear functionals $k_i: H^q[a,b] \to I\!R$, $i = 1,\dots,n$, for g when this information $z = (z_1,\dots,z_n)$ is imprecise, i.e. $|k_i g - z_i| \le \varepsilon_i$, $\varepsilon_i \ge 0$, $i = 1,\dots,n$.

As approximation of g we are looking for the solution of the minimization problem

$$\int_a^b (f^{(q)}(t))^2\, dt \longrightarrow \min_{f\in H^q[a,b],\ \ |k_i f - z_i|\le\varepsilon_i,\ \ \varepsilon_i\ge 0,\ \ i=1,\dots,n}. \tag{1}$$

It is known that the solution of this problem is a polynomial spline from the space $S = \{\ s \in H^q[a,b] :\ \int_a^b (s^{(q)}(t)x^{(q)}(t))\, dt = 0\ for\ all\ x \in \mathrm{Ker}A\ \}$, where $A = (k_1,\dots,k_n)$.

Under the conditions of the existence and uniqueness of this solution of the smoothing problem (1) the methods for its construction are suggested and investigated for some concrete problems which were considered according to the information z on a mesh $a = t_0 < t_1 < ... < t_{m+1} = b$:

1) for the problem of smoothing of the values of g at knots t_i, $i = 1,\dots,n$, (see [1, 2]), i.e. when $k_i g = g(t_i)$, $i = 1,\dots,n$, $n = m$.

2) for the problem of approximation of given histogram by smoothing histosplines (see [3]), i.e. when $k_i g = \frac{1}{t_i - t_{i-1}} \int_{t_{i-1}}^{t_i} g(t)dt, i = 1,\dots,n$, $n = m+1$.

3) for the problem of smoothing of an unknown function g by Hermite splines, i.e. when $k_i g = g(t_i)$, $k_{i+m} g = g'(t_i)$, $i = 1,\dots,n$, $n = m$.

References

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