ON STABILITY OF CLASSES OF LIPSCHITZ MAPPINGS.

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By the well-known results of Yu.G. Reshetnyak, F.John and others, classes of conformal and isometric mappings are stable (see [1]). A.P. Kopylov suggested general ways to the stability problems of classes of mappings which he called the concepts of ξ- and ω-stability [2]. In essence, stability of a class $\mathfrak{G}$ means that local proximity of a mapping $f : \Delta \subset \mathbb{R}^n \to \mathbb{R}^m$ to the mappings of the class $\mathfrak{G}$ implies global proximity of f to them in the C-norm.

In our talk we give overview of progress in these stability theories and present some new results.

Recall that a class $\mathfrak{G}$ of Lipschitz functions is called *ω-stable* [2] if there exists a function $\sigma : [0, +\infty) \to [0, +\infty)$ such that (1) $\sigma(\varepsilon) \to \sigma(0) = 0$ as $\varepsilon \to 0$; (2) the inequality $\omega(f, \mathfrak{G}) \le \sigma(\varepsilon)$ holds for every function $f : \Delta \to \mathbb{R}$ of a domain $\Delta \subset \mathbb{R}^n$ such that $\Omega(f, \mathfrak{G}) \le \varepsilon$.

Here $\omega(f, \mathfrak{G}) = \sup_{B\subset\Delta} \omega_B(f, \mathfrak{G})$, $\Omega(f, \mathfrak{G}) = \sup_{x\in\Delta}\{\overline{\lim\limits_{r\to 0}}\, \omega_{B(x,r)}(f, \mathfrak{G})\}$, where $B{=}B(x, r)$ is a ball in Δ and

$$\omega_B(f, \mathfrak{G}) = \inf_{g:B\to\mathbb{R},\ g\in\mathfrak{G}} \{r^{-1} \sup_{y\in B} |f(y) - g(y)|\}.$$

The functionals $\omega(\cdot, \mathfrak{G})$ and $\Omega(\cdot, \mathfrak{G})$ are referred to as the functionals of *global* and *local proximity* to the class $\mathfrak{G}$.

One of our result is as follows. Let $G \subset \mathbb{R}^n$ be a compact set and $\delta : \mathbb{R}^n \setminus G \to \mathbb{R}$ be a positive function such that $0 < \delta(x) \le \operatorname{dist}(x, G)$. Let $\mathfrak{Z}^+_\delta(G)$ be the class of all Lipschitz functions $g : \Delta \to \mathbb{R}$ defined on domains $\Delta \subset \mathbb{R}^n$ such that

$$g'(x) \in G \text{ a.e.} \tag{1}$$

and

$$\forall a \in \mathbb{R}^n \setminus G\ \forall B(x, r) \subset \Delta \sup_{y\in B(x,r)} \big(g(y) - g(x) - a(y - x)\big) \ge \delta(a)r.$$

Theorem. The class $\mathfrak{Z}^+_\delta(G)$ is ω-stable.

Examples. If $n = 1$ and $G \subset \mathbb{R}$ is a totally disconnected set then the class $\mathfrak{Z}^+_\delta(G)$ coincides with the class of convex functions satisfying inclusion (1) (see [3]). (In this case $\mathfrak{Z}^+_\delta(G)$ does't depend on the function δ.) In general case the class $\mathfrak{Z}^+_\delta(G)$ always contains the class of convex functions satisfying (1).

References.

[1] Reshetnyak Yu. G. Stability Theorems in Geometry and Analysis // Nauka, Novosibirsk, 1996

[2] Kopylov A. P. On stability of isometric mappings // Sibirsk. Mat. Zh., 25, p. 132–144 (1984).

[3] Korobkov M.V. On Stability of a Class of Convex Functions // Progress in Analysis. Proceedings of the 3rd International Isaac Congress (Berlin, Germany, 20-25 August 2001) Vol.1, P.207-213, World Scientific Publishing, 2003