ABSTRACT

ON THE RIEMANN EXTENSIONS OF SPACE-TIMES WITH VANISHING CURVATURE INVARIANTS

Dryuma Valery

Institute of Mathematics and Informatics AS Moldova, Kishinev

E-mail: valery@dryuma.com; cainar@mail.md

The properties of the eight-dimensional Ricci-flat (${}^8R_{ik}=0$) Riemannian spaces D^8 with the metric

$$ds^2=-2\Gamma^k_{ij}(x^l)\Psi_k dx^i dx^j+2d\Psi_k dx^k \tag{1}$$

having the geodesic equations in form

$$\ddot{x}^k+\Gamma^k_{ij}\dot{x}^i\dot{x}^j=0,\qquad \frac{\delta^2\Psi_k}{ds^2}+R^l_{kji}\dot{x}^j\dot{x}^i\Psi_l=0, \tag{2}$$

are studied. Here Ψ_k are additional coordinates ($k=1...4$) and

$$\frac{\delta\Psi_k}{ds}=\frac{d\Psi_k}{ds}-\Gamma^l_{jk}\Psi_l\frac{dx^j}{ds},$$

$\Gamma^k_{ij}=\Gamma^k_{ij}(x^l)$ are the coefficients of affine connection of four-dimensional Ricci-flat (${}^4R_{ik}=0$) spaces-time

$$ds^2=dx^2+dy^2+2dzdt+H(x,y,z)dz^2. \tag{3}$$

The metric (3) is example of the metric of pp-waves with vanishing scalar invariants of curvature tensor R^i_{ikl}.

The system (2,2) for additional coordinates Ψ_k has the form of linear 4×4 matrix system of the second order ODE's

$$\frac{d^2\vec{\Psi}}{ds^2}+A(s)\frac{d\vec{\Psi}}{ds}+B(s)\vec{\Psi}=0 \tag{4}$$

and their properties with help of the invariants of the 4×4 matrix-function $E=B-\frac{1}{2}\frac{dA}{ds}-\frac{1}{4}A^2$ are studied. The invariants of the system (4) and its Darboux-transformation for investigation of the properties of the spaces-times (3) are used.

The examples of the eight-dimensional extensions of the Einstein spaces with nonvanishing scalar invariants can be also considered. In particular the properties of the Riemann extension of the Schwarzschild space-time have been studied.

References

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