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Soliton-Type Asymptotics and Scattering for Coupled Maxwell-Lorentz Equations

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Consider a relativistic charge coupled to a Maxwell field. If $q(t) \in \mathbb{R}^3$ is the position of the charge at a time t, the coupled Maxwell-Lorentz equations read

$$\begin{aligned} &\dot{E}(x,t) = \nabla \wedge B(x,t) - \rho(x-q(t))\dot{q}(t), && \nabla \cdot E(x,t) = \rho(x-q(t)), \\ &\dot{B}(x,t) = -\nabla \wedge E(x,t), && \nabla \cdot B(x,t) = 0, \\ &\dot{q}(t) = p(t)/(1+p^2(t))^{1/2} =: v(t), && \dot{p}(t) = \int [E(x,t) + \dot{q}(t) \wedge B(x,t)]\rho(x-q(t))\, d^3x. \end{aligned} \quad (1)$$

Here $E(x,t)$, $B(x,t)$ is the Maxwell field, ρ is the charge distribution on which we impose some simple regularity conditions and either *smallness condition* (S) or *Wiener condition* (W),

$$\|\rho\|_{L^2} \ll 1 \quad \text{(S)}; \qquad \hat{\rho}(k) = \int e^{-ikx}\rho(x)d^3x \neq 0 \text{ for } k \in \mathbb{R}^3. \quad \text{(W)}$$

The system (1) admits *solitons*, i.e. the solutions of type $(E_v(x-vt), B_v(x-vt), vt, p_v)$ with a $v \in \mathbb{R}^3$, $|v| < 1$; and a conjecture occurs that they describe long-time asymptotics of all finite energy solutions. Let us make a survey of recent results obtained in collaboration with A. Komech, N. Mauser, and H. Spohn.

In [1] the existence of dynamics on a suitable phase space is established, together with the important estimate $|\dot{q}(t)| \le \overline{v} < 1$, $t \in \mathbb{R}$. The main results of [1], under the condition (W) and a certain power decay of initial data at infinity, are the relaxation of the acceleration, $\ddot{q}(t) \to 0$ *as* $t \to \pm\infty$ and the preliminary result on soliton-type asymptotics, $(E(q(t)+x,t), B(q(t)+x,t)) - (E_{v(t)}(x), B_{v(t)}(x)) \to 0$ as $t \to \pm\infty$. The convergence here is in *local energy seminorms* and $(E_{v(t)}(x), B_{v(t)}(x))$ is a *co-moving* soliton (not a fixed one).

In [2], under the condition (S) and the same decay of initial data, we prove that

$$\dot{q}(t) \to v_\pm, \quad (2)$$

$$(E,B)(x+q(t),t) \to (E_{v_\pm}, B_{v_\pm})(x), \quad (3)$$

$$(E,B)(x,t) \to (E_{v(t)}, B_{v(t)})(x,t) + U(t)F_\pm, \quad (4)$$

as $t \to \pm\infty$; here $U(t)$ is the free Maxwell group and $F_\pm$ are *scattering states*. In all three cases the convergence is of a power rate. The asymptotics (3) hold in local energy seminorms and the asymptotics (4) which mean *scattering behavior* of solutions, hold in *global energy norm*. The results are extended to the case of compact-supported external electric and magnetic potentials inserted to the system (1).

In [3] we establish *orbital stability* of solitons and prove the asymptotics (2) and (3), this time under the condition (W) assuming a bit stronger power decay of initial data then in [2]. The convergence (3) holds in local energy seminorms.

Few words about the methods used. The proof in [1] is based on the Wiener Tauberian theorem. In [2] we use a non-autonomous integral inequality method. In [3] we analyse the Hamiltonian structure of the system, develop a version of Hamiltonian formalism for the perturbed system, and apply the strong Huygen's principle.

References

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