

Section number: 14

Yuriy. V. Tokovyy and Andriy V. Rychahivskyy

Pidstryhach Institute for Applied Problems of Mechanics and Mathematics  
NAS of Ukraine

e-mail: [tokovyy@iapmm.lviv.ua](mailto:tokovyy@iapmm.lviv.ua)

## **Method of Direct Integration of Differential Equations for Two- and Three-Dimensional Elasticity and Thermoelasticity Problems**

By means of the prof. V. Vihak method based on direct integration of differential equations for quasi-static elasticity and thermoelasticity problems, the solutions for main two- and three-dimensional problems in the regions without corner points as well as of the plane problem in a rectangle are successfully constructed without any use of auxiliary harmonic or biharmonic functions. Application of the method to solving elasticity problems in the case of orthotropic material is also demonstrated.

The method is based on integration of elasticity equations, determination of relations between the stress tensor components, and selection of the so-called governing stresses. Consequently, the key integro-differential equations are written down for the governing stresses. To solve them, a method for the variables separation is proposed. After determination of stresses the displacements are found by integration of the Cauchy relations.

It is proved for several regions that the original compatibility equations in terms of strains are of integro-differential type. They can be reduced to the well-known differential compatibility equations only if corresponding co-ordination conditions are satisfied.

It is demonstrated for three-dimensional problems of mechanics of deformable solids that there are only three differential compatibility equations in terms of strains. These last naturally coincide with separate three Saint-Venant's compatibility equations, depending on which three of the six Cauchy relations have been selected as the governing ones for finding displacements. On the basis of the three compatibility equations in terms of strains the corresponding equations in terms of stresses are written down. Together with equilibrium conditions they constitute a complete set of equations in terms of stresses for three-dimensional elasticity and thermoelasticity problems.

As a result of integration of the equilibrium and Cauchy equations, the integral equilibrium conditions for stresses as well as compatibility conditions for strains and displacements are deduced.

MSC (1991): 74B05

**Key words:** direct integration, thermoelasticity problems, equilibrium and compatibility equations, integral equations.