INTEGRODIFFERENTIAL HYPERBOLIC EQUATIONS AND THEIR APPLICATIONS TO 2-D ROTATIONAL FLUID FLOWS

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A number of problems of theoretical hydrodynamics can be deduced to the integrodifferential equations which can be written as

$$U_t+A(U)\langle U_x\rangle=0. \qquad (1)$$

Here $U(t,x,y)$ – desired vector and A is a nonlocal matrix operator acting over the variable y. Theoretical analysis of such systems is based on a generalization of the concept of hyperbolicity and characteristics for equations with operator coefficients suggested by V.M. Teshukov [1,2]. This paper deals with mathematical models of type (1) describing in long-wave approximation plane-parallel rotational motion of a perfect homogeneous and two-layer stratified fluid with free surface in channels with solid and elastic walls. Necessary and sufficient conditions for hyperbolicity of the systems for homogeneous fluid flows with monotonic velocity depth profiles are formulated. The propagation velocities of the characteristics of the systems under study and the characteristic forms of these systems are calculated. Based on the generalization of energy integrals approach, the uniqueness of the Cauchy problem is established for sufficiently smooth initial data. New classes of exact solutions of equations of motion are constructed. The existence of simple waves (solutions of the form $U=U(a(t,x),y))$ continuously attached to a given steady shear flow $U=U(y)$ is proved. Numerical simulations of 2-D rotational fluid flows are performed. In order to describe discontinuous rotational flows, the integrodifferential equations of motion are written in a special conservation forms and jumps conditions are derived. Some numerical methods, in particular, non-oscillatory central schemes developed earlier for differential conservation laws, are shown to be applicable in numerical study of more complicated integrodifferential systems.

REFERENCES

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