Systems of orthogonal polynomials defined by hypergeometric type equations

Nicolae Cotfas

University of Bucharest, Romania, **E-mail:** ncotfas@yahoo.com
http://fpcm5.fizica.unibuc.ro/~ncotfas

A hypergeometric type equation satisfying certain conditions defines either a finite or an infinite system of orthogonal polynomials. We present in a unified and explicit way all these systems of orthogonal polynomials, the associated special functions and the corresponding raising/lowering operators [2,3,4]. Our systematic study recovers a number of earlier results in a natural unified way and also leads to new findings. Certain results well-known in the case of infinite systems of orthogonal polynomials [5] are extended to the case of finite systems, less studied to our knowledge.

The polynomials we analyse are directly related to the bound-state eigenfunctions of some important Schrödinger equations (Pöschl-Teller, Scarf, Morse, etc) and allow us to analyse these equations together, in a unified formalism. Our systems of polynomials can be expressed in terms of the classical ones [5], but in certain cases one has to consider the classical polynomials outside the interval where they are orthogonal or for complex values of parameters [1]. Generally, our results can not be obtained in a simple way from those concerning the classical polynomials, and we think that our general approach is useful.

References

[1] F. Cooper, A. Khare, U. Sukhatme: Supersymmetry and quantum mechanics, *Phys. Rep.* **251** (1995) 267-85

[2] N. Cotfas: Shape invariance, raising and lowering operators in hypergeometric type equations, *J. Phys.A: Math. Gen.* **35** (2002) 9355-65

[3] N. Cotfas: Special functions, raising and lowering operators, *Inst. Phys. Conf. Ser.* **173** (2003) 649-52

[4] N. Cotfas: Systems of orthogonal polynomials defined by hypergeometric type equations with application to quantum mechanics, *Central European Journal of Physics* (2004), accepted for publication

[5] A. F. Nikiforov and V. B. Uvarov: *Special Functions of Mathematical Physics*, Birkhäuser, Basel (1988).