A theory of the destabilization paradox in non-conservative systems

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In the middle of the last century Ziegler [1] studying the stability of a double pendulum loaded by a follower force came to the unexpected conclusion that the critical load of the non-conservative system with vanishingly small dissipation is considerably lower than in the case when dissipation is completely absent. The analytical description of this phenomenon called *the destabilization paradox* was recognized as one of the main theoretical challenges in the non-conservative stability theory [2] and attracted much attention in the world literature [3, 4]. However, the questions provoked by the destabilization paradox have not yet been answered in the general form. In the present paper a new theory qualitatively and quantitatively describing the paradoxical behavior of general non-conservative systems due to small dissipative and gyroscopic forces is developed. The problem is investigated by the approach based on the multiparameter sensitivity analysis of multiple eigenvalues. The behavior of eigenvalues of the system in the complex plane is analytically described and interpreted. Approximations of the stabilization domain in the space of the system parameters are obtained. An explicit asymptotic expression for the critical load as function of dissipation and gyroscopic parameters allowing to calculate a jump in the critical load is derived. The classical problems by Ziegler [1] and Herrmann and Jong [3] considered as mechanical applications demonstrate the efficiency of the theory.

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