CLASSES OF MODULES

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ABSTRACT. The last decade has brought into the first line and developed a study of certain classes of modules, such as natural classes, prenatural classes or open classes. All these are formally defined as classes of modules closed under some of the following: submodules, direct sums, direct products, homomorphic images, isomorphic copies, extensions, injective hulls. We intend to present a short survey on some recent developments on this topic and, furthermore, to consider and study certain associated classes. Thus to some non-empty classes $\mathcal{C}$ closed under isomorphic copies we associate classes $\mathcal{A}_\mathcal{C}$ in the following ways: (*i*) if $\mathcal{C}$ is closed under submodules and extensions, then $\mathcal{A}_\mathcal{C}$ consists of the modules that are not in $\mathcal{C}$, but all their proper homomorphic images are in $\mathcal{C}$; (*ii*) if $\mathcal{C}$ is closed under homomorphic images and extensions, then $\mathcal{A}_\mathcal{C}$ consists of the modules that are not in $\mathcal{C}$, but all their proper submodules are in $\mathcal{C}$.

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