Exceptional naturally graded p-filiform Lie algebras *

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Abstract

The classification of nilpotent Lie algebras is very controversial due to the complexity of the involved computations. So far, it is only known up to dimension 7 [4].

The study of some important families selected according their properties seems crucial.

The classification of filiform Lie algebras (maximal nilindex) is obtained up to dimension 11. It is based on descomposition due to Khakimdjanov [4].

The interesting family of p-filiform Lie algebras is a large family of Lie algebras, comprising the filiform ones as a particular class. Their classification for $p \geq n-4$ is obtained in arbitrary finite dimension n [1]. However, the classification of p-filiform Lie algebras of dimension n (when $p = 1$ or near 1) seems very difficult.

The knowledge of naturally graded algebras of a particular Lie algebra class offers valuable information about the structure of that class. Thus, we know the "skeleton" of laws of Lie algebras associated to each naturally graded algebra. Also the geometric description of the characteristically nilpotent filiform Lie algebras can be obtained by using the naturally graded filiform algebras. In addition, these algebras offer important information about the irreductible components of the variety of nilpotent Lie algebras.

Vergne has obtained the classification in the case filiform [5] in arbitrary finite dimension n. For sufficiently high n and any admissible value of p the results are a generalization of Vergne's [3], [2]. However, as to little dimensions, there are algebras which do not appear in high dimensions. We call them exceptional algebras.

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