THE ENDOMORPHISMS SEMIGROUPS OF FREE GROUPS

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Let $F(X)$ — is a free group over alphabet X (not more then countable),
$$EF(X) = \operatorname{End} F(X),$$
$$EF_0(X) = \{\varphi \in EF(X) | \operatorname{Im} \varphi \text{ is cyclic}\}.$$

Theorem of density. *Let S be an arbitrary semigroup. The conditions (1), (2) are equivalent:*

(1) $S \cong EF(X)$;

(2) *the semigroup S is the maximal dense extension (in the sense of [1, 2]) of its ideal $D \cong EF_0(X)$.*

For ideal $EF_0(X)$ the description in terms of Rees matrix construction is obtained.

The structure properties of the semigroup $EF(X)$ are characterized by the help of embedding of its in some matrix construction that arise as realization of the double-semigroup package ([3]).

The semiretractions ([4]) of the ideal $EF_0(X)$ saving Rees construction are described.

The similar technique for investigation of some free products of groups is applicated.

References

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