On $\mathbf{Z}$-equivalence of two forms associated with a quiver and one Roiter's conjecture

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Let Γ is a quiver (i.e. oriented graph) with a finite number points $\{1,\ldots,n\}$ (vertices) and arrows (edges) without loops and oriented cycles. It is well known in representation theory so-called Tits quadratic form $T_\Gamma(x)$ of Γ (introduced by P.Gabriel [1]) defined as $T_\Gamma(x) = T_\Gamma(x_1,\ldots,x_n) = \sum_{i=1}^n x_i^2 - \sum_{i\to j} x_i x_j$ (where second sum is taken on the all arrows of a quiver Γ). Also it is naturally to call corresponding to $T_\Gamma(x)$ non-symmetric bilinear form $T_\Gamma(x,y) = T_\Gamma(x_1,\ldots,x_n,y_1,\ldots,y_n) = \sum_{i=1}^n x_i y_i - \sum_{i\to j} x_i y_j$ as non-symmetrical bilinear Tits form of Γ. On the other side, we can consider the another quadratic (resp. bilinear) form obtaining by natural way from the quiver Γ (see [3]). We call quadratic (resp. non-symmetric bilinear) pathes form of the Γ and denote by P_Γ the following form: $P_\Gamma(x) = \sum_{i,j=1}^n \lambda_{ij} x_i x_j$ (resp. $P_\Gamma(x,y) = \sum_{i,j=1}^n \lambda_{ij} x_i y_j$), where λ_{ij} is the number of pathes from i to j in Γ.

Theorem 1. Quadratic pathes form $P_\Gamma(x)$ and Tits form $T_\Gamma(x)$ is canonically $\mathbf{Z}$-equivalent. Non-symmetrical bilinear pathes form $P_\Gamma(x,y)$ of a quiver Γ is canonically $\mathbf{Z}$-equivalent to non-symmetrical Tits form $T_{\Gamma^0}(x,y)$ of the quiver Γ^0 which is anti-isomorphic to the quiver Γ.

As a **corollary**, in the case of finite partially ordered set (poset) $M = \{m_1,\ldots,m_n\}$ with simply connected Hasse graph Γ we obtain the canonical $\mathbf{Z}$-equivalence between the characteristic form $\chi_M(x_1,\ldots,x_n)$ of partially ordered relation (introduced by A.V.Roiter), i.e. form $\sum_{m_i\le m_j} x_i x_j$, and the Tits form T_Γ of corresponding to M Hasse graph $\Gamma = \Gamma_M$ (see [3]).

One of the criterions for finite (or tame) representation type of poset may be formulated in the terms of properties of χ_M, exactly, in the terms [2] of the norm of relation $||(M,\le)|| = \min\{\chi_M(x) | x \in K\}$ where $K = \{x | \sum_{i=1}^n x_i = 1 \textit{ and the all } x_i \ge 0\}$. Namely, $(M,\le)$ has finite (resp., tame) representation type iff $||(M,\le)|| > \frac{1}{4}$ (resp., iff $||(M,\le)|| \ge \frac{1}{4}$) (see [2]).

Following to [2] the posets $(M,\le)$ is said to be *P-exact* iff its norm may be achived only on exact vectors (i.e. vectors with non-zero coordinates) from the simplex K. For example, posets from classical lists of critical and hypercritical posets (given by M.Kleiner and L.Nazarova respectively) are P-exact.

Also, as in [2] the poset S is said to be *fence* if S is a union of t nonintersective chains (i.e full ordered subposets) $Z_1,\ldots,Z_t$, where $||Z_i|| \ge 2$ $(i = \overline{1,t})$, $t > 1$; $\min(Z_i) < \max(Z_{i+1})$, $i = \overline{1,t-1}$ and there are no other comparisons between elements of different chains.

In [2] A.V.Roiter found all fencies (which he called *uniform*) which may be P-exact and formulated conjecture, asserting that finite poset is P-exact iff it is disjoint union (cardinaly sum) of chains and (or) some uniform fensies.

In this talk we prove, that if connected poset M is P-exact, then χ_M is positive definite (**Theorem 2**) and corresponding to M Hasse graph $\Gamma = \Gamma_M$ is simply connected (**Theorem 3**). Hence, due to **corollary** of **Theorem 1**, in this case the Tits form T_Γ is also positive definite and therefore Γ is one of the Dynkin's graphs with ordinary edges. We show, that really Γ is $\mathbf{A}_n$ and M is a chain or a fence. So, we obtain the full and explicit description of P-exact posets and prove ([3]) the Roiter's conjecture ([2]) about their structure.

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Kiev Taras Shevchenko National University, e-mail: zeldich@mail.ru ; section 02 and(or) 06 Classificacion 15AG3; 16G20; Key words: pathes form and Tits form for a quiver; characteristic form of partially ordered relation; Hasse graph of partially ordered set (poset); P-exact poset.