Section number: 05 – Topology

Author: Michael N. Shmatkov, Sobolev Institute of Mathematics, Russia, e-mail: smn@math.nsc.ru

Title of the poster: Counting cyclic covering of 3-manifolds

Text of abstract: We consider a specific variety of the famous Hurwitz enumeration problem [1]. Recall that a regular covering is called cyclic if its covering transformations group is isomorphic to the cyclic group $\mathbb{Z}_n$. Using certain algebraic technique based on the results of G.A. Jones [2] we derive formulas for counting the number $\mathsf{N}_{\mathcal{B}}(n)$ of non-equivalent cyclic n-sheeted coverings over an arbitrary Seifert manifold $\mathcal{B}$ specified by the set of its invariants $(b;(\varepsilon,g);(\alpha_1,\beta_1),\ldots,(\alpha_r,\beta_r))$ [3], where b is a bundle number, ε is one of the six possible types of Seifert manifolds, g is the genus of a base surface, and (α_j,β_j) are the invariant of exceptional fibres of $\mathcal{B}$. The main idea of our method is to reduce the original problem to counting the number of solutions of several systems of congruences modulo n. The latter is made with the help of specific combinatorial and number theoretic methods involving generating functions, von Sterneck function, Ramanujan sum. As a result we obtain the exact formulas allowing one to count directly the number of coverings mentioned as a function of a positive integer n. The formulas are of the form

$$\mathsf{N}_{\mathcal{B}}(n)=\frac{1}{\varphi(n)}\sum_{m|n}\mathsf{H}_{\mathcal{B}}(m)\mu(n/m),$$

where $\varphi(n)$ is the Euler function, $\mu(n)$ is the Möbius function, and $\mathsf{H}_{\mathcal{B}}(n)$ is treated as the number of solutions of the specific system of congruences modulo n corresponding to the bundle $\mathcal{B}$. The multiplicativity of the latter functions makes calculations more convenient.

[1] *Hurwitz A.,* Über Riemann'sche Elächen mit gegeben Verzweigungspunkten, Math. Ann., Bd. 39, 1–61 (1891).

[2] *Jones G.A.,* Counting subgroups of non-euclidean crystallographic groups, Math. Scand., V. 84, 23–39 (1999).

[3] *Orlik P.,* Seifert manifolds, Lecture Notes in Mathematics (291), Berlin–Heidelberg–New York, Springer–Verlag, 1972.

1991 MSC: 05E99, 14E20, 55R05, 57M12

Key words: cyclic covering, fundamental group, Seifert manifold, multiplicative function