Multi-agent interaction of complex systems in restricted resources environment *

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Abstract

In this paper we study the properties of complex systems based on replicator dynamics. Vector field analysis is used to reveal new properties of the dynamical system. Spectral problem has been considered and Lyaponov function has been derived from the algebraic properties of the system under consideration. New mathematical model with multi-agent interaction is developed and general properties of such model are discussed. The over-all dynamics can be understood by means of mathematical modeling of replicator equations and corresponding multi-agent system. The dynamics of multi agent system depends crucially on the interaction between agents and it is quite different from the dynamics of corresponding independent agents.

keywords: replicator dynamics, Hamiltonian systems, gradient dynamical system, multi-agent interaction, Lyapunov function, complex system.

1 Abstract

How natural systems function and organize themselves is a very attractive and challenging problem. This problem is extremely complex studying living organisms involving a great number of biomacromolecules and cells performing a very specific role in organisms. In the same time the functioning of living system as a whole as a rule doesn't have common features with functioning of its parts. The problem of studying social system like organization of market economy is also extremely complex because behavior of the systems parts is quite different from the behavior of the system as a whole and the occurrence, growth, and development of the market is due to self-organization processes. In order to

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understand a little bit surrounding world a different mathematical models has been introduced for grasping some of the features of the systems existing in nature. One of such approaches is based on mathematical model of multiagent systems which consist from many interacting agents. In such systems agents are goal seeking entities with their own goals and the ability to control their own behavior in pursuing these goals. A great interest to such a system is connected with their nonlinear complex behavior unpredictable from analyzing behavior of the agents. Due to this reason investigation of multiagent systems dynamics is important interdisciplinary area of research that appears in modern sciences. One of fundamental problem in multiagent systems is conflict resolution. Indeed agents as the entities being capable of acting in the environment to satisfy their desire try to optimize their goals. If the agents demand for shared but limited resource a certain conflict arises because each of agents tries to survive. The interaction between agents leads to an adaptive behavior of any entity and as a result system as a whole possessing nontrivial evolution being the subject of research in various fields of science[1, 2, 3].

The problem under consideration is closely connected with fundamental problem of how complex multi-agent systems widely met in nature can adapt to changes in the environment when there is no centralized control in the system. For complex "living" system such problem has been considered in [4, 5, 6]. Contrary replicator dynamics arises if the agents have to deal with conflicting goals and the behavior of such systems is quite different from the adaptation problem considered in[4, 6, 5].. If two different agents are located in the same corner point it will arise additional force which destroy such distribution. Such results confirms by simulation simple model of four interacting agents. So, we can formulate next statement.

Statement. If free agents passes the specialization their cooperative behavior will tend to diversification.

All the niches (strategies) are occupied. As a rule we get stationary distribution of the agents. The matter is that the intercorrelation between agents leads to the additional friction and depression of the dynamics. In this case diverse distribution of the agents is stable according to small perturbation. Such properties that the system posses the diverse behavior has been already discussed in literature [13, 14] for another multi-agent model.Conclusions

A new dynamical model for the multi-agent systems has been developed, based on a general algebraic properties of agent dynamics. The motivation was the total probability of each agent to use a strategies is equal to one and as a result we obtained that the correlation between agents strategies determines the dynamics of the agent strategies. The average pair correlation of the agents determines the system dynamics as a whole.

We have presented only first results of the introduced model and general pattern of such multiagent dynamics is still far from being well understood. Nevertheless we can single out several main results which can be summarized from this study.

First, we investigated a general properties of the replicator dynamics, namely, from vector field analyzes found expression for Lyaponov function a and have

studied spectral properties of the system. Second, we derived new mathematical description of multiagent interaction based on replicator dynamics. Third, we made a computer simulation of the simple cases of the derived model and obtained nontrivial synergetic behavior of the system. the made investigation helped us in understanding the complex system dynamics being a simple model of the surrounding world.

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