Consider the following controlled Hamiltonian system

$$\dot{q}_1 = \frac{\partial H}{\partial p_1}, \quad \dot{q}_2 = \frac{\partial H}{\partial p_2}, \quad \dot{p}_1 = \tau, \quad \dot{p}_2 = -\frac{\partial H}{\partial q_2} \tag{1}$$

where $q_1 \in R^1, q_2 \in R^n$ are generalized coordinates, $p_1 \in R^1, p_2 \in R^n$ are momenta (generalized impulses), $H = H(q_2, p_1, p_2)$ is Hamiltonian, and τ is a controlling generalized force (torque). This system under the condition $\tau = 0$ has two classical first integrals H and p_1.

Sometimes the desired motion may be described by two desired first integrals H_d and p_{1d}. To design the stabilizing control let us consider the following Lyapunov function

$$V = \frac{\alpha}{2}(H - H_d)^2 + \frac{\beta}{2}(p_1 - p_{1d})^2 \tag{2}$$

where α and β are positive constants. This type of Lyapunov functions was considered earlier . It is easy to check that

$$\dot{V} = \alpha(H - H_d)\dot{q}_1\tau + \beta(p_1 - p_{1d})\tau$$

If we choose as control

$$\tau = -F(\alpha(H - H_d)\dot{q}_1 + \beta(p_1 - p_{1d})) \tag{3}$$

where F is a continuous, strictly increasing, possibly, bounded function with the condition $F(0) = 0$, then $\dot{V} \leq 0$ and in some cases it may be applied La-Salle principle of invariance. There is another way to investigate the closed loop system. For the deviations of energy and first generalized impulse the following differential equations hold

$$(H - H_d)^{\cdot} = \tau\dot{q}, \quad (p_1 - p_{1d})^{\cdot} = \tau \tag{4}$$

The Lyapunov function is positive definite in reference to the variables $H - H_d$ and $p_1 - p_{1d}$; so we may apply to the extended system (1), (3), (4) the theorem about asymptotic stability in reference to the part of variables [2].

We consider the following examples of application of this general idea: rotating body and beam [1], permanent rotation of rigid body, motion of controlled material point around gravitational center.

References

[1] Coron J.-M., d'Andrea-Novel B. (1998) Stabilization of rotating body beam without damping. *IEEE Tr. AC* **43**: 608-618.

[2] Rumyantsev V.V., Oziraner A.S. (1987) *Stability and Stabilization in Reference to the Part of Variables.* Moscow, Nauka (in Russian). Also: *J. Appl. Math. Mech.* (1973) **37**, iss.4.