This paper deals with the stabilizability of the system described by the following partial differential equation

$$\begin{cases} \frac{\partial z}{\partial t} = \alpha \frac{\partial^2 z}{\partial x^2} + \gamma z + b(x)\,u(t) & (x,t) \in \,]0,1[\, \times \,]0,\infty[\\ z(x,0) = z_0(x) & x \in \,]0,1[\\ z(0,t) = z(1,t) & t \in \,]0,\infty[\end{cases} \tag{1}$$

where $b \in L_2(0,1)$.

This equation can be rewritten in the abstract form with state space $H = L_2(0,1)$, where

$$\dot{x}(t) = Ax(t) + \widetilde{b}u(t), x(0) = x_0 \tag{2}$$

$$Au = \alpha \frac{\partial^2 u}{\partial x^2} + \gamma u \tag{3}$$

for u in the domain of A given by

$$D(A) = \{u \in H : \ddot{u} \in H \text{ and } u(0) = u(1) = 0\} \tag{4}$$

The system (2) is open-loop stabilizable if the solution $x(.)$ is square integrable for some $u(.)$.This concept of stabilizability is weaker than the concept of stabilizing the system (2) by a bounded feedback, meanning under what conditions on (2) does there exists a $F \in L(H, \mathbb{R})$ such that the solution of the closed system

$$\dot{x}(t) = (A + \widetilde{b}F)x(t), x(0) = x_0 \tag{5}$$

converges to zero exponentially, for every $x_0 \in H$.

If H is finite-dimensional, these two concepts are shown to be equivalent. In the infinite dimensional case the equivalence was proved, see [1], for the bounded state operator case. In [1] Zwart considered some classes of infinite dimensional systems with unbounded state operator and proved the equivalence under the condition that $\widetilde{b} \in D(A)$. In this work we study the above mentioned problem for a class of infinite dimensional linear systems: those having a single input but for which the state operator A is self adjoint with compact resolvant. Necessary and sufficient conditions for the system to be exponentially stabilizable will be given. These conditions are given in terms of the eigenvalues of the infinitesimal generator and the Fourier coefficients of input operator. Then the result on equivalence between these concepts of stabilizability is stated and proved.

References

[1] H.J.Zwart;Open loop stabilizability, a research note, proceedings of the IFAC conference on Distributed Parameter Systems, Perpignan, France, 26/29 June 1989, pp.505-509.