Global stability and periodic solutions of a class of differential delay equations

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Differential delay equations appear as mathematical models of a wide variety of real life phenomena in physiology, mathematical biology, laser optics, economics, and other fields [1, 3, 4, 5]. This talk discusses the global stability properties and the existence of periodic solutions of scalar differential delay equations of the form

$$x'(t) = \begin{vmatrix} f_1(x(t-\tau)) & g_1(x(t)) \\ f_2(x(t-\tau)) & g_2(x(t)) \end{vmatrix} = f_1(x(t-\tau))g_2(x(t)) - f_2(x(t-\tau))g_1(x(t)). \qquad (1)$$

Functions f_i and $g_i, i = 1, 2$, are assumed to be continuous and such that the equation has a unique equilibrium. Equation (1) includes several important partial cases coming from applications.

For the global asymptotic stability two types of sufficient conditions are established: (i) delay independent, and (ii) conditions involving the size τ of the delay. Delay independent conditions make use of the global stability in the limiting (as $\tau \to \infty$) difference equation $f_1(x_n)g_2(x_{n+1}) = f_2(x_n)g_1(x_{n+1})$: the latter always implying the global stability in the differential equation for all values of the delay $\tau \geq 0$. The delay dependent conditions involve the global attractivity in specially constructed one-dimensional maps (difference equations) that include the nonlinearities f_i and g_i, and the delay τ. The global stality results are established in [2]

Sufficient conditions are given for equation (1) to have a nontrivial periodic solution. This typically happens when the unique equilibrium is unstable, the equation possesses the negative feedback, and there is a uniform boundedness of all solutions.

Relevance to existing models from applications is also discussed.

References

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