A Generalization and a New Proof of the Kalman-Yakubovich-Popov Lemma in a Hilbert Space.

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The Kalman-Yakubovich-Popov (KYP) lemma [1, 2, 3] is a cornerstone of system theory. It has applications in stability theory of nonlinear systems, in optimal control, in stochastic realization theory, and in other areas. We extend the most general known formulation of the KYP lemma in a Hilbert space, which is given in [4].

The KYP lemma claims the equivalence of the following three statements:

(1) There exists a solution of the Lur'e equation. (In the non-singular case the Lur'e equation is equivalent to the algebraic Riccati equation.)

(2) There exists a solution of the Lur'e inequality.

(3) The so called frequency condition is fulfilled. (This condition is expressed in terms of some function that maps complex plane to the space of selfadjoint operators in a Hilbert space. Condition holds if this function has positive semi-definite values on the imaginary axis.)

In our generalization of the KYP lemma the frequency condition is fulfilled on an arbitrary straight line or circle on the complex plane. The statements (1) and (2) are modified accordingly. In the case of unit circle this result is the infinite-dimensional version of the Kalman-Szego lemma [5].

To prove the lemma we consider an auxiliary extremum problem in the space of selfadjoint trace class operators. It turns out that each of statements (1)–(3) is a necessary and sufficient condition for the value to be finite in this extremum problem. We treat the auxiliary problem using a new Fenchel duality theorem for a class of extremum problems defined on a cone in a vector lattice.

It is shown that the statement (1) is a necessary condition for the value to be finite in the extremum problem which is dual to the considered auxiliary one. The statement (3) is a sufficient condition for the primal problem to have a finite value. The obvious implications (1)⇒(2)⇒(3) complete the proof.

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