ON THE GLOBAL CONTROL OF THE MORPHOLOGICAL INSTABILITY OF SOLIDIFICATION FRONT

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The solidification of binary substances is subject to the morphological instability. The original problem includes the diffusion equation of the solute in the liquid phase and the boundary conditions. We use a simple model of a long wave instability in the case of a small segregation coefficient, which was adopted by G. Sivashinsky to describe the nonlinear development of the morphological instability.

A.J. Bernoff and A.L.Bertozzi have proved that the nonlinear development of disturbances is characterized by a subcritical instability and blow-up.

We analyzed the possibility to control the long-wave morphological instability by means of adjusting the applied temperature gradient and the pulling velocity as functionals of control function $U(\Phi(x,\tau)) : G(U(\Phi(x,\tau)))$ and $V(U(\Phi(x,\tau)))$ respectively.

In the case when the temperature gradient and the pulling velocity are modulated, the coefficient of viscosity in the Sivashinsky equation is modulated:

$$F_\tau + \beta F_{\xi\xi\xi\xi} - [d + U(\tau)]F_{\xi\xi} - \frac{1}{2}((F)^2)_{\xi\xi} + \kappa F = 0, \qquad (1)$$

where $d = -1$ in the supercritical case, $d = 1$ in the subcritical case.

As a control function, we shall take

$$U(\tau) = -P \min_{\xi} F(\xi, \tau), \qquad (2)$$

where P is a positive constant.

The role of the control is enhancing the "viscosity" when negative peaks of $F(\xi,\tau)$ appeared (which can lead to a blow-up).

We considered the analytically tractable case $\kappa \to 0$ (small segregation).

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The corresponding solutions of the equation are easily found analytically in terms of elliptical functions with modulus $0 < q \leq 1$.

If $q = 1$ the solution corresponds to a *solitary wave* one.
If $0 < q < 0.988$ we have *supercritical* solution $(d = -1)$, if $0.988 < q < 1$ the solution corresponds to the *subcritical* case $(d = 1)$.

By numerical simulation, we get some solutions for nonlinear equation.