A parabolic equation with drift

$$\frac{\partial u}{\partial t} = \frac{1}{2}\,\Delta\, u + \langle B, \operatorname{grad} u\rangle,$$

is considered on a complete simply connected n-dimension Riemannian manifold $\mathcal{M}$ with non-positive curvature, known as Cartan—Hadamard manifold. In the equation the drift field $B : \mathcal{M} \to T\mathcal{M}$ is a vector field on the manifold $\mathcal{M}$ and Laplacian Δ is concerned with the Riemannian metrics, i. e. is Laplace—Beltrami operator.

The first boundary problem for this equation is solved with potential method. In order to apply this method, existing a double-layer potential gap is proven on a submanifold included into the manifold $\mathcal{M}$ [1]. Cartan—Hadamard manifold exhibits behavior similar to $\mathbb{R}^n$.

A fundamental solution for the parabolic equation with drift is proposed to be constructed by the perturbation method similar to parametrix one [2]. Also the fundamental solution estimates are obtained with different assumptions concerning the drift field [3]. Two groups of assumption are examined: strong one means integrability of the drift field norm, its first and second covariant derivatives; and weak one means bounded drift field, bounded first and second covariant derivatives of the field.

A logarithmic gradient of the fundamental solution is shown to be represented as sum of two vector fields: known one and bounded one [4]. The boundedness of the latter field is proven under strong assumptions for the drift field by maximum principle.

References

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