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Optimal Control and Approximation of Some Differential Parabolic Inclusion

The paper is concerned with optimization problem and its approximation for the cost functional

$$J(y,u) = \int_0^T \|y(t)\|^2_{L^2(\Omega)} dt + \int_0^T \|u(t)\|^2_{L^2(\Omega)} dt$$

Where y is a solution of parabolic inclusion

$$\frac{\partial y(t,x)}{\partial t} - \sum_{ij=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial y(t,x)}{\partial x_j}\right) + a_0(x)y(t,x) + \partial\chi(y(t,x)) \ni u(t,x) \text{ a.e. } Q$$

$$y(0,x) = y_0 \text{ a.e. } \Omega$$

$\Omega \subset \mathbb{R}^n$ is a set of C^0 class with boundary Γ $a_0, a_{ij} \in C^\infty(\Omega)$ for $i,j = 1,2,\ldots,n$ and

$$\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \geq \alpha \sum_{i=1}^{n} \xi_i^2 \quad \forall\, \xi_i, \xi_j \in \mathbb{R},$$

$$a_0(x) \geq \alpha \text{ for certain } \alpha > 0,$$

$$[a_{ij}(x)]_{1\leq i,j\leq n} \text{ is symmetric matrix}$$

$$\chi(y(t)) = \begin{cases} 0 & \text{for} \quad y(t) \in C \\ +\infty & \text{for} \quad y(t) \in H_0^1(\Omega) \setminus C. \end{cases}$$

The set C is any convex closed subset of $H_0^1(\Omega)$, $\operatorname{int} C \neq \emptyset$ and $y_0 \in \operatorname{int} C$.

We derive some results on the existence of optimal solutions. We describe the Galerkin approximation and we demonstrate existence of the weak condensation points of a set of solutions of the approximate optimization problems. Each of these points is a solution of the initial optimization.

References

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