

# NONLINEAR PHENOMENA

in Stockholm:

*Kinetic Meets Dispersive*

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Book of Abstracts



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## Håkan Andreasson

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### ON THE EXISTENCE AND STRUCTURE OF STATIC AND STATIONARY SOLUTIONS OF THE EINSTEIN-VLASOV SYSTEM

#### **Abstract:**

The present status on the existence and structure of static and stationary solutions of the Einstein-Vlasov system is reviewed. The structure of spherically symmetric static solutions is quite well understood and a number of their features will be discussed. I will in particular present a more recent result on the existence of static massless solutions. These solutions are models of self-gravitating photon shells, or spherically symmetric geons. For axially symmetric solutions much less is known but an existence result will be described. Furthermore, I will present numerical results of axially symmetric rotating stationary solutions. In particular I will discuss two different sequences of toroidal solutions which contain ergoregions. These solutions either approach an extreme Kerr black hole in the limit or the solutions have the property that the geometry becomes conical in the limit and such solutions may provide models of cosmic strings.

# Anudeep Kumar Arora

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## SINGULARITIES AND GLOBAL SOLUTIONS IN THE SCHRÖDINGER HARTREE EQUATION

### **Abstract:**

We consider a nonlinear Schrödinger type equation with nonlocal nonlinearity, of a convolution type, called the generalized Hartree equation. In the focusing case we investigate global behavior of solutions and formation of stable singularities. In the inter-critical regime we first obtain a dichotomy for global vs finite time existing solutions exhibiting two methods of obtaining scattering: one via Kenig-Merle concentration - compactness and another one is using Dodson-Murphy approach via Morawetz on Tao's scattering criteria. Next, we investigate stable blow-up solutions in a critical regime and describe the blow-up dynamics, which is similar to NLS. This work is a part of the PhD dissertation under the supervision of Svetlana Roudenko.

**Alexander V. Bobylev**

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ON THE VLASOV-POISSON-LANDAU KINETIC EQUATION FOR  
PLASMA

**Abstract:**

We consider some properties of the Vlasov-Poisson-Landau kinetic equations for the case when the typical length is the mean free path, not the Debye radius. What are the limiting equations in this case? Are they well-posed? How to describe the corresponding necessary conditions of transition to the limit? How important are collisional effects? These and similar questions are the main subject of the talk. The talk is based on joint work with Irina Potapenko

## Luccas Campos

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### BLOW-UP, SCATTERING AND CONCENTRATION FOR SOLUTIONS TO INHOMOGENEOUS NONLINEAR SCHRÖDINGER EQUATION

#### Abstract:

We consider the initial value problem for the focusing inhomogeneous nonlinear Schrödinger (INLS) equation:

$$\begin{cases} i\partial_t u + \Delta u + |x|^{-b}|u|^{p-1}u = 0, & t > 0, x \in \mathbb{R}^N \\ u(x, 0) = u_0(x), \end{cases}$$

where  $u = u(x, t)$  is a complex-valued function in space-time  $\mathbb{R}^N \times \mathbb{R}$  and  $b > 0$ .

We will talk about some results regarding the long-time behavior of solutions to INLS.

1. First, a dichotomy between blow-up and scattering above the mass-energy threshold;
2. Then a result about scattering below the threshold that improves known results, using tools by Tao, Dodson and Murphy;
3. Finally, we show some results about concentration for finite-time blow-up solutions in the case  $0 \leq s_c < 1$ .

The main tools for this purpose are, for the first item, the ideas in [1] and sharp interpolation inequalities, as in [2]. For the second item, the ideas in [3] and [4]. The third item is the extension to INLS of ideas in [5] and [6].

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**Michele Correggi**

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## ON THE AVERAGE-FIELD FUNCTIONAL FOR ANYONS

**Abstract:**

We discuss the average-field approximation for a trapped gas of non-interacting anyons in the quasi-bosonic regime. In the homogeneous case, i.e., for a confinement to a bounded region, we prove that the energy in the regime of large statistics parameter, i.e., for “less-bosonic” anyons, is independent of boundary conditions and of the shape of the domain. When a non-trivial trapping potential is present, we derive a local density approximation in terms of a Thomas-Fermi-like model. We also discuss some recent numerical simulations and open questions mostly related to the vortex structure of the minimizer.

Joint work with R. Duboscq (Toulouse), D. Lundholm (Stockholm) and N. Rougerie (Grenoble).



**Magdalena Czubak**

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**ALMOST SURE BOUNDEDNESS OF ITERATES FOR SEMILINEAR  
WAVE EQUATIONS**

**Abstract:**

We study nonlinear wave equations in 2D with quadratic derivative nonlinearities with random initial data in the energy space. We obtain a uniform time interval on which the Picard iterates of all orders are almost surely bounded. This is in contrast to the deterministic results.

## Esther Daus

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### MULTI-SPECIES CROSS-DIFFUSION POPULATION MODELS: EXISTENCE OF SOLUTIONS AND DERIVATION FROM UNDERLYING PARTICLE MODELS

#### **Abstract:**

In the first part of this talk, we focus on the proof of the existence of global-in-time weak solutions to reaction-cross-diffusion systems for an arbitrary number of competing population species. In the case of linear transition rates, the model extends the two-species population model of Shigesada, Kawasaki, and Teramoto. The existence proof is based on a refined entropy method and a new approximation scheme. Global existence follows under a detailed balance or weak cross-diffusion condition, where the detailed balance condition is related to the symmetry of the mobility matrix, which mirrors Onsager's principle in thermodynamics.

The second part of the talk links at the formal level the entropy structure of the cross-diffusion system satisfying the detailed balance condition with the entropy structure of a reversible microscopic many-particle Markov process on a discretised space. Moreover, we present a very recent proof of a rigorous mean-field limit from a stochastic particle model to a cross diffusion model. These results are based on a joint work with Xiuqing Chen and Ansgar Juengel, a joint work with Helge Dietert and Laurent Desvillettes, and a joint work with Li Chen and Ansgar Juengel.

## Emanuela Giacomelli

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### CORNER CONTRIBUTION TO SURFACE SUPERCONDUCTIVITY

**Abstract:**

We consider an extreme type-II superconducting wire with non-smooth cross section, i.e., with one or more corners at the boundary, in the framework of the Ginzburg-Landau theory. We prove the existence of an interval of values of the applied field where the energy is not affected by the presence of corners to leading order. To isolate the contributions to the energy density due to the corners we then introduce a new effective problem. The explicit expression of the effective energy is yet to be found, but we formulate a conjecture on it based on the behavior for almost flat angles. Based on a joint work with Michele Correggi.

## Jonatan Lenells

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### ASYMPTOTICS FOR THE SINE-GORDON EQUATION IN THE QUARTER PLANE

#### **Abstract:**

I will discuss an initial-boundary value problem for the sine-Gordon equation posed in the quarter plane. After describing how this problem can be analyzed via Riemann-Hilbert techniques, I will present asymptotic formulas for the solution. Assuming that the initial and boundary values approach integer multiples of  $2\pi$  for large  $x$  and  $t$ , we will see that the long-time behavior is described by three asymptotic sectors. Of particular interest are formulas for the topological charge of the solution and for the interaction between the asymptotic solitons and the radiation background.

## Douglas Lundholm

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### AVERAGE-FIELD ANYONS FROM QUANTUM DE FINETTI

**Abstract:**

Quantum systems confined to planar geometries may exhibit effective particles with unusual statistics, called anyons. These can be modeled as identical particles (bosons or fermions) in 2D with magnetic flux attached to them. In this talk I will discuss the derivation, using a quantum de Finetti theorem, of the average-field approximation for anyons, in which the resulting magnetic field is replaced by that of the mean distribution of the particles. In a certain limit this turns out to be a correct description and the particles then behave like independent, identically distributed bosons interacting via a self-consistent magnetic field. This is joint work with Nicolas Rougerie (Grenoble).

## Hans Lundmark

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### PEAKON SOLUTIONS OF THE NOVIKOV AND GENG–XUE EQUATIONS

#### **Abstract:**

I will first give an introduction to the subject of integrable partial differential equations admitting peaked soliton solutions, or peakons for short. The most well-known such equation is the Camassa–Holm shallow water wave equation, but there are several others, such as the Degasperis–Procesi equation and two of its close mathematical relatives which I will focus on in particular, namely the Novikov equation and the Geng–Xue equation. All these equations are similar in many respects, but they also have interesting differences, for example regarding how regular the solutions need to be, and how solutions can be continued past a singularity where some kind of breakdown occurs.

Explicit formulas for the multipeakon solutions are known, and with their aid one can for example study in detail the kind of wave-breaking that takes place when a positive-amplitude peakon collides with a negative-amplitude antipeakon. This is particularly interesting for the Novikov equation, whose peakon-antipeakon solutions display a much wider array of behaviours than usual, including the possibility of several peakons and antipeakons travelling together in breather-like clusters.

The Geng–Xue equation is interesting in a different way. It is a two-component system, with many possible inequivalent configurations depending on the order in which the peakons appear in the two components. The solution formulas describing an arbitrary configuration are very intricate and have been derived only recently, relying not only on the usual inverse spectral techniques, but also (crucially) on a certain limiting procedure for turning peakons into “ghostpeakons” with amplitude zero.

This talk is based on joint works with Jacek Szmigielski, Marcus Kardell and Budor Shuaib.

## Julian Mauersberger

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### THE HYPERBOLIC ERNST EQUATION IN A TRIANGULAR DOMAIN

**Abstract:**

In Einstein's theory of relativity, the interaction of two plane gravitational waves can be described mathematically by a Goursat problem for the hyperbolic Ernst equation in a triangular domain. In this talk, we show how to use the integrable structure of the hyperbolic Ernst equation to present the solution of the Goursat problem in terms of a corresponding Riemann–Hilbert problem. Our results treat uniqueness, existence and regularity, and a representation formula of the solution. Moreover, we determine the—for the application to gravitational waves crucial—boundary behavior of the solution near the boundary of the triangle. This is joint work with Jonatan Lenells.

## Dana Mendelson

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### ALMOST SURE WELL-POSEDNESS FOR SOME NONLINEAR DISPERSIVE EQUATIONS

#### **Abstract:**

In this talk, I will discuss several problems on nonlinear wave and dispersive equations with random initial data, including the energy critical nonlinear wave and Schroedinger equations, and derivative nonlinear wave equations. I will present several almost sure well-posedness and scattering results for these equations and contrast the ways in which random data techniques can be exploited in these different contexts.



**Anne Millet**

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**RANDOM FIELD GLOBAL SOLUTION TO A STOCHASTIC WAVE EQUATION WITH SUPERLINEAR COEFFICIENTS.**

**Abstract:**

We prove the existence and uniqueness of a random field global solution to a wave equation subject to a random perturbation. This perturbation is driven by the space-time white noise in dimension 1, and by a noise white in time with a spatial correlation described in terms of a density - such as a Riesz kernel - in dimensions 2 and 3. The drift and diffusion coefficients are supposed to have superlinear growth. This is joint work with M. Sanz-Solé.

**Alessia Nota**

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## HOMOENERGETIC SOLUTIONS FOR THE BOLTZMANN EQUATION

### **Abstract:**

We consider a particular class of solutions of the Boltzmann equation, known as homoenergetic solutions, which are useful to describe the dynamics of Boltzmann gases under shear, expansion or compression in nonequilibrium situations. While their well posedness theory has many similarities with the theory of homogeneous solutions of the Boltzmann equation, their long time asymptotics differs completely, due to the fact that these solutions describe far-from-equilibrium phenomena. Indeed, the long time asymptotics cannot always be described by Maxwellian distributions. For several collision kernels the asymptotics of homoenergetic solutions is given by particle distributions which do not satisfy the detailed balance condition.

In this talk I will describe different possible long time asymptotics of homoenergetic solutions of the Boltzmann equation as well as some open problems in this direction.

(Joint work with Richard D. James and Juan J.L. Velázquez).

## Natasa Pavlovic

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### ON THE BOLTZMANN EQUATION VIA DISPERSIVE TOOLS

**Abstract:** Motivated by works on rigorous derivation of nonlinear Schrödinger equations from quantum many particle systems, in this talk we will focus on the close connection between the Boltzmann equation and nonlinear Schrödinger equations in the density matrix formulation. This connection has been recognized implicitly for some time, but we wish to make it quite explicit and to the best of our knowledge this is the first time such an explicit connection has been established. The talk is based on joint works with Thomas Chen and Ryan Denlinger.

Ronald Quirchmayr

ON THE SPECTRAL PROBLEM ASSOCIATED WITH THE TIME-PERIODIC NLS

**Abstract:**

According to its Lax pair formulation, the nonlinear Schrödinger equation (NLS) can be expressed as the compatibility condition of two linear ordinary differential equations with an analytic dependence on a complex parameter. The first of these equations—often referred to as the  $x$ -part of the Lax pair—can be rewritten as an eigenvalue problem for a Zakharov-Shabat operator. The spectral analysis of this operator is crucial for the solution of the initial value problem for NLS via inverse scattering techniques. For space-periodic solutions, this leads to the existence of a Birkhoff normal form, which beautifully exhibits the structure of NLS as an infinite-dimensional completely integrable system. In this talk we present several aspects of a recent joint work with Jonatan Lenells, where we take a first few steps towards developing an analogous picture for time-periodic solutions by performing a spectral analysis of the  $t$ -part of the Lax pair with a periodic potential.

## Svetlana Roudenko

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### ON SPECTRAL PROPERTIES FOR BLOW-UP SOLUTIONS AND SOLITON STABILITY IN DISPERSIVE EQUATIONS

**Abstract:**

When studying the blow-up dynamics and soliton stability in the NLS-type and KdV-type equations, we encounter spectral property questions, arising from linearization, or from the virial-type arguments. We will mention examples of the  $L^2$ -critical blow-up dynamics, spectral properties and its consequences in the NLS and the Hartree equations, and then describe a more challenging situation due to the absence of a simple virial identity in Zakharov-Kuznetsov (ZK) equation, a higher dimensional generalization of the KdV equation. We prove the blow-up in the 2d cubic ZK equation (an  $L^2$ -critical case) and discuss the asymptotic stability in the  $L^2$ -subcritical cases.

**Chiara Saffirio**

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FROM THE MANY-BODY QUANTUM DYNAMICS TO THE VLASOV EQUATION.

**Abstract:**

We review some results on the joint mean-field and semiclassical limit of the  $N$ -body Schrödinger dynamics leading to the Vlasov equation, which is a model in kinetic theory for charged or gravitating particles. The results we present include the case of singular interactions and provide explicit estimates on the convergence rate, using the Hartree-Fock theory for interacting fermions as a bridge between many-body and Vlasov dynamics.

## Stanley Snelson

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### CONDITIONAL REGULARITY FOR THE INHOMOGENEOUS LANDAU EQUATION

**Abstract:**

This talk will describe an ongoing program to find conditions under which solutions to the spatially inhomogeneous Landau equation are smooth and can be continued forward past a given time. We will focus particularly on recent joint work with C. Henderson and A. Tarfulea, which used probabilistic methods to establish self-generating pointwise lower bounds, implying a new continuation criterion. Time permitting, we will discuss possible strategies for improving this criterion further.

## Maja Taskovic

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### ON THE RELATIVISTIC LANDAU EQUATION

#### **Abstract:**

The Landau equation, derived in 1936 by Landau, models a dilute hot plasma in which fast moving particles interact via Coulomb interactions. It does not include the effects of Einstein's theory of special relativity. However, when particle velocities are close to the speed of light, which happens frequently in a hot plasma, then relativistic effects become important. A model that captures these effects, the relativistic Landau equation, was derived by Budker and Beliaev in 1956.

We study the Cauchy problem for the spatially homogeneous relativistic Landau equation with Coulomb interactions. The difficulty of the problem lies in the extreme complexity of the kernel of the collision operator. We present a new decomposition of this kernel. This is then used to prove the global entropy dissipation estimate, propagation of polynomial moments for a weak solution, and the existence of a true weak solution for a large class of initial data. This is joint work with Robert M. Strain.



**Kai Yang**

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**BLOW-UP DYNAMICS IN THE CRITICAL AND SUPERCRITICAL  
NONLINEAR SCHRÖDINGER TYPE EQUATIONS**

**Abstract:**

We present our numerical results for the stable blow-up solutions in the  $L^2$ -critical NLS equation, proving the log-log blow-up dynamics up to the dimension 12 via the Spectral property, and then in the  $L^2$ -supercritical NLS, exhibiting profiles and a square root blow-up rate with a different self-similar profile which is not in  $L^2$ . Then, we will show the similar numerical results that we obtained for the generalized Hartree equation (the NLS equation with non-local potential).