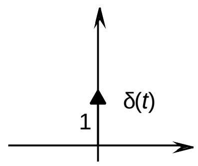
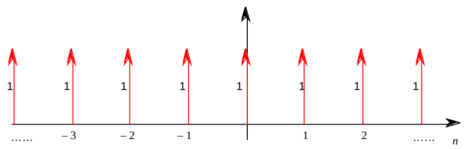
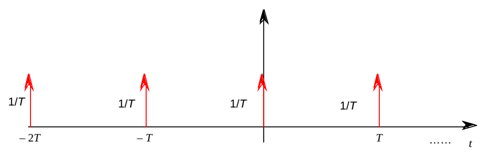
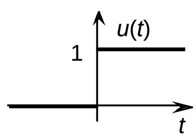
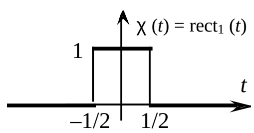
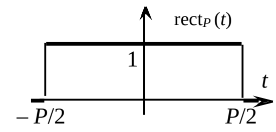
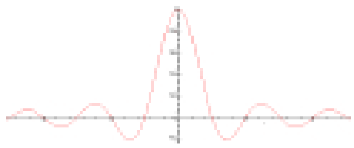
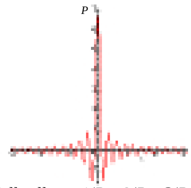
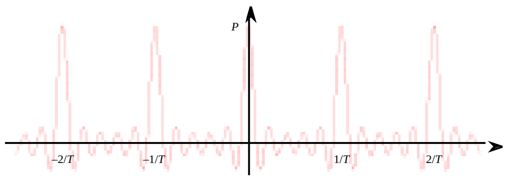
Funktionslexikon

OW: = Oppenheim-Willsky, Hj: = Hjalmarsson et.al., F.matr = Föreläsningsmaterial
Transformvariabel i OW: ω. Transformvariabel i HJ och F.matr: *f*.

$$2\pi f = \omega, \quad df = \frac{1}{2\pi}d\omega$$

Beteckning	litterturhänvisning	Viktigare relationer	Grafer
$\delta(t)$ (Deltafunktionen)	OW 1.4.2, Hj 4.2.1, F.matr 2	$y(t)\cdot\delta(t-a) = y(a)\cdot\delta(t-a)$ $\int_{-\infty}^{\infty} y(t)\cdot\delta(t-a)\,dt = y(a)$ $\delta(at) = \frac{1}{a}\delta(t), a > 0$ $\delta(t) = \int_{-\infty}^{\infty} e^{2\pi j f t}\,df$ $\delta(t) \overset{F}{\leftrightarrow} 1$ $1 \overset{F}{\leftrightarrow} \delta(f)$	
$\delta'(t)$,	F.matr 2	$\int_{-\infty}^{\infty} y(t)\cdot\delta'(t-a)\,dt = -y'(a)$ $\delta'(t) \overset{F}{\leftrightarrow} 2\pi j f$ $t \overset{F}{\leftrightarrow} -\delta'(f)/(2\pi j f)$	
$\Delta(t) = \sum_{n=-\infty}^{\infty}\delta(t-n)$ (Pulståg)	F.matr 2, 3	$\Delta(t) \overset{F}{\leftrightarrow} \Delta(f)$ $\Delta(t) = \sum_{n=-\infty}^{\infty} e^{2\pi j n t}$ Sampling av $y(t)$ i heltalspunkterna: $y(t)\,\Delta(t) = \sum_{n=-\infty}^{\infty} y(n)\delta(t-n)$ 1-periodisk fortsättning av $y(t)$: $y(t) * \Delta(t) = \sum_{n=-\infty}^{\infty} y(t-n)$	
$\frac{1}{T}\Delta(t/T) = \sum_{n=-\infty}^{\infty}\delta(t-nT)$		$\frac{1}{T}\Delta(t/T) \overset{F}{\leftrightarrow} \Delta(fT)$ $\sum_{n=-\infty}^{\infty}\delta(t-nT) \overset{F}{\leftrightarrow} \sum_{n=-\infty}^{\infty}\delta(fT-n)$ Sampling av $y(t)$ i punkterna nT: $y(t)\cdot\sum_{n=-\infty}^{\infty}\delta(t-nT) = \sum_{n=-\infty}^{\infty} y(n)\delta(t-nT)$ P-periodisk fortsättning av $y(t)$: $y(t) * \sum_{n=-\infty}^{\infty}\delta(t-nT) = \sum_{n=-\infty}^{\infty} y(t-nP)$	
$u(t) = 1$ om $t > 0$, $= 0$ annars. (Unit-stepfunction, Heavisides funktion, $H(t)$)	OW 1.4.2, Hj 4.2.1, F.matr 2 §2.5	$u'(t) = \delta(t)$	
$\text{rect}_1(t) = \chi(t) =$ $=1$ om $\lvert t\rvert < 1/2$, $= 0$ annars	Hj 4.2.2, F.matr 5	$\text{rect}_1(t) \overset{F}{\leftrightarrow} \text{sinc}\, f$	

$\mathrm{rect}_P(t) = \mathrm{rect}_1(t/P)$	Hj 4.2.2	Skalad variant av $\mathrm{rect}_1(t)$ $\mathrm{rect}_P(t) \xrightarrow{F} P \,\mathrm{sinc}\, Pf$	
$\mathrm{sinc}\, t = \dfrac{\sin \pi t}{\pi t}$ (Sinus cardinalis, ”sinken“)	OW 4.1.3, Hj 4.2.2, F.matr 5	$\mathrm{sinc}\, t \xrightarrow{F} \mathrm{rect}_1(f)$ $\lim_{P\to\infty} P\,\mathrm{sinc}\,(Pt) = \delta(t)$	
$d_P(t) = P\,\mathrm{sinc}\,(Pt)$	Hj 4.2.2	Skalad variant av sinc t. $P\,\mathrm{sinc}\,(Pt) \xrightarrow{F} \mathrm{rect}_P(f)$ $\lim_{P\to\infty} d_P(t) = \delta(t)$	 Nollställen: ±1/P, ±2/P, ±3/P, …
$S_{T,P}(t) = T\dfrac{\sin \pi t P}{\sin \pi t T}$ $= T \sum_{n=-(N-1)/2}^{(N-1)2} e^{2\pi j t T n}$ ($N = P/T$ är udda heltal)	Hj 5.2 - 3, F.matr 2	$\sum_{n=-\infty}^{\infty} \delta(t-nT)\cdot \mathrm{rect}_P(t) \xrightarrow{F} S_{T,P}(f)$ $S_{T,P}(f) = \sum_{n=-\infty}^{\infty} \delta(Tf-n) * d_P(f)$ $\lim_{P\to\infty} S_{T,P}(t) = \sum_{n=-\infty}^{\infty} \delta(t-n/T)$ $\lim_{T\to 0} S_{T,P}(t) = P\,\mathrm{sinc}\,(Pt) = d_p(t)$ $\lim_{P\to\infty} \left(\lim_{T\to 0} S_{T,P}(t)\right) = \delta(t)$	
$\hat{Y}_P(f)$	Hj 6.4	FT av trunkerad signal $y(t)$: $y(t)\cdot \mathrm{rect}_P(t) \xrightarrow{F} \hat{Y}_P(f) = Y(f) * P\,\mathrm{sinc}\, Pf$	
$\hat{Y}_T(f)$ Obs. Ej samma som $\hat{Y}_P(f)$ med variabeln P utbytt mot T!	Hj 6.5	FT av samplad signal $y(t)$: $y(t)\cdot \sum_{n=-\infty}^{\infty} \delta(t-nT) \xrightarrow{F} \frac{1}{T}\hat{Y}_T(f)$ $\hat{Y}_T(f) = Y(f) * \sum_{n=-\infty}^{\infty} \delta(f-n/T)$ (Poissons summationsformel)	
$\hat{Y}_{T,P}(f)$	Hj 6.2-3	FT av samplad och trunkerad signal: $y(t)\cdot \sum_{n=-\infty}^{\infty} \delta(t-nT)\cdot \mathrm{rect}_P(t)$ $\xrightarrow{F} \hat{Y}_{T,P}(f) =$ $Y(f) * \sum_{n=-\infty}^{\infty} \delta(Tf-n) * d_P(f) =$ $Y(f) * S_{T,P}(f)$	