Krzysztof Burdzy

Open problems: Suppose that D is a bounded open set in R^n and $k > 0$ is a fixed integer. Consider partitions $\{D_j\}_{1\le j\le k}$ of D, i.e., families of disjoint open sets $D_j \subset D$ such that $D \setminus \bigcup_{1\le j\le k} D_j$ has an empty interior. Let $\lambda_j > 0$ denote the first eigenvalue for the Laplacian with Dirichlet boundary conditions in D_j. A particle model considered in mathematical physics literature ([BHIM] K. Burdzy, R. Hołyst, D. Ingerman and P. March "Configurational transition in a Fleming-Viot-type model and probabilistic interpretation of Laplacian eigenfunctions" *J. Phys. A* **29**, 1996, 2633–2642; [CBH] O. Cybulski, V. Babin and R. Hołyst "Minimization of the Renyi entropy production in the space-partitioning process" *Phys. Rev. E* **71**, 046130, 2005) gives rise to the following problems.

(i) Does there exist a partition which minimizes $\sum_{1\le j\le k} \lambda_j$? The answer is positive—this follows from the results of D. Bucur, G. Buttazzo and A. Henrot "Existence results for some optimal partition problems" *Adv. Math. Sci. Appl.* **8** (1998), no. 2, 571–579.

(ii) The partition that minimizes $\sum_{1\le j\le k} \lambda_j$ is not always unique, for example, if D is a ball. Characterize domains using geometric conditions where the partition minimizing $\sum_{1\le j\le k} \lambda_j$ is unique.

(iii) Characterize pairs (D, k) such that the partition that minimizes $\sum_{1\le j\le k} \lambda_j$ forms the nodal domains of an eigenfunction of the Laplacian. See [BHIM] and [CBH] for specific conjectures when D is a rectangle.

(iv) Is it true that for a fixed D and k sufficiently large, the partition that minimizes $\sum_{1\le j\le k} \lambda_j$ looks like honeycomb, i.e., most of the sets D_j have an approximately hexagonal shape?