# An inverse spectral problem coming from seismology
## (problem session at the Matrei conference July 9, 2005).

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Let $q \in C^\infty(]-\infty, 0], ]0, \infty[)$ be so that $q(x) \equiv 1$ for $x << 0$ and $q(x)$ is not everywhere $\geq 1$.

Let $h$ be a $> 0$ number and $L_h$ be the following family of self-adjoint operator:

$$L_h = -h^2 \partial_x q \partial_x + q$$

with Neumann boundary condition at $x = 0$.

Let $\sigma_q(h) \subset ]\inf q, 1[$ be the discrete spectrum of $L_h$. We ask the following question:

**is it possible to recover $q$ from the spectra $\sigma_q(h)$ supposed to be known for all $h > 0$ ?**

Using semi-classical asymptotics $h \to 0$ given be the Weyl law, one can check that the answer is yes if $q$ is decaying.

A more difficult question would be the same problem, but assuming only the knowledge of the spectra of $L_h$ for $h \in [h_0, h_1]$ with $h_0 > 0$.

# References

[1] Yves Colin de Verdière, Hearing the noise of the earth: mathematical models, preprint 2005.

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