

10.3.14.

$$\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x - y^2 \\ -y + xy \end{pmatrix} = \mathbf{g}(\mathbf{X})$$

$$\text{Kritiska punkter : } \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x - y^2 \\ -y + xy \end{pmatrix} .$$

$$2x - y^2 = 0$$

$$-y + xy = -y(1 - x) = 0$$

$$a) \quad y = 0 \quad x = 0, \quad (x, y) = (0, 0)$$

$$b) \quad x = 1 \quad y = \pm\sqrt{2}, \quad (x, y) = (1, \pm\sqrt{2})$$

$$\mathbf{g}(\mathbf{X}) = \begin{pmatrix} 2 & -2y \\ y & -1+x \end{pmatrix}$$

$$\mathbf{g}(0,0) = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}, \quad \Delta = -2 < 0.$$

$(0,0)$ är en sadelpunkt.

$$\mathbf{g}(1, \sqrt{2}) = \begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 0 \end{pmatrix}$$

$$\Delta = 4 > 0, \quad \tau = 2 > 0, \quad \tau^2 - 4\Delta = -12 < 0.$$

$(1, \sqrt{2})$ är en instabil spiral.

$$\mathbf{g}(1, -\sqrt{2}) = \begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 0 \end{pmatrix}$$

$$= 4 > 0, \quad \tau = 2 > 0, \quad \tau^2 - 4 = -12 < 0.$$

$(1, -\sqrt{2})$ är en instabil spiral.

Alternativ framställning.

$(0,0)$

$$\begin{array}{r} x \\ y \end{array} = \begin{array}{r} 2x \\ -y \end{array} + \begin{array}{r} -y^2 \\ xy \end{array} = \begin{array}{r} 2 \\ 0 \end{array} \begin{array}{r} 0 \\ -1 \end{array} \begin{array}{r} x \\ y \end{array} + \begin{array}{r} -y^2 \\ xy \end{array}$$

$$\mathbf{A} = \begin{array}{cc} 2 & 0 \\ 0 & -1 \end{array}, \quad \mathbf{0} = (\mathbf{A} - \lambda \mathbf{I})\mathbf{v} = \begin{array}{cc} 2 - \lambda & 0 \\ 0 & -1 - \lambda \end{array} \mathbf{v}$$

$$\lambda_1 = 2, \quad \mathbf{v}_1 = \begin{array}{c} 1 \\ 0 \end{array}, \quad \lambda_2 = -1, \quad \mathbf{v}_2 = \begin{array}{c} 0 \\ 1 \end{array}.$$

$$(1, \sqrt{2})$$

$$\text{Sätt :} \quad \begin{array}{ll} u = x - 1 & u = x \\ v = y - \sqrt{2} & v = y \end{array} \cdot$$

$$\begin{array}{l} u \\ v \end{array} = \begin{array}{l} 2(u + 1) - (v + \sqrt{2})^2 \\ -(v + \sqrt{2}) + (u + 1)(v + \sqrt{2}) \end{array}$$

$$\begin{array}{l} u \\ v \end{array} = \begin{array}{l} 2u - v^2 - 2v\sqrt{2} \\ uv + u\sqrt{2} \end{array}$$

$$\begin{matrix} u \\ v \end{matrix} = \begin{matrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 0 \end{matrix} \begin{matrix} u \\ v \end{matrix} + \begin{matrix} -v^2 \\ uv \end{matrix}$$

$$\mathbf{B} = \begin{matrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 0 \end{matrix}$$

$$0 = \det(\mathbf{B} - \lambda\mathbf{I}) = \lambda^2 - 2\lambda + 4 = (\lambda - 1)^2 + 3.$$

$$\lambda = 1 \pm i\sqrt{3} \quad , \quad \text{Instabil spiral.}$$

$$(1, -\sqrt{2})$$

Sätt :

$$\begin{array}{ll} u = x - 1 & u = x \\ v = y + \sqrt{2} & v = y \end{array} \cdot$$

$$\begin{array}{l} u \\ v \end{array} = \begin{array}{l} 2(u + 1) - (v - \sqrt{2})^2 \\ -(v - \sqrt{2}) + (u + 1)(v - \sqrt{2}) \end{array}$$

$$\begin{array}{l} u \\ v \end{array} = \begin{array}{l} 2u - v^2 + 2v\sqrt{2} \\ uv - u\sqrt{2} \end{array}$$

$$\begin{array}{l} u \\ v \end{array} = \begin{array}{cc} 2u + 2v\sqrt{2} & -v^2 \\ -u\sqrt{2} & uv \end{array} +$$

$$\begin{array}{l} u \\ v \end{array} = \begin{array}{cc} 2 & 2\sqrt{2} \\ -\sqrt{2} & 0 \end{array} \begin{array}{l} u \\ v \end{array} + \begin{array}{l} -v^2 \\ uv \end{array}$$

$$\mathbf{C} = \begin{array}{cc} 2 & 2\sqrt{2} \\ -\sqrt{2} & 0 \end{array}$$

$$0 = \det(\mathbf{C} - \lambda\mathbf{I}) = \lambda^2 - 2\lambda + 4 = (\lambda - 1)^2 + 3.$$

$$\lambda = 1 \pm i\sqrt{3} \quad , \quad \text{Instabil spiral.}$$