

10.3.18.

$$\begin{aligned} x &= x(1 - x^2 - 3y^2) \\ y &= y(3 - x^2 - 3y^2) \end{aligned} = \mathbf{g}(\mathbf{X})$$

Kritiska punkter:

$$\begin{aligned} x(1 - x^2 - 3y^2) \\ y(3 - x^2 - 3y^2) \end{aligned} = \mathbf{g}(\mathbf{X}) = \mathbf{0}$$

$$(0,0), (0, \pm 1), (\pm 1, 0)$$

$$\mathbf{g}(\mathbf{X}) = \begin{pmatrix} 1 - 3x^2 - 3y^2 & -6xy \\ -2xy & 3 - x^2 - 9y^2 \end{pmatrix}$$

$$\mathbf{g}(0,0) = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \quad \tau = 4 > 0$$

$$\tau^2 - 4 = 4 > 0$$

$(0,0)$ är instabil nod.

$$\mathbf{g}(0, \pm 1) = \begin{pmatrix} -2 & 0 \\ 0 & -6 \end{pmatrix} \quad \tau = -8 < 0$$

$$\tau^2 - 4 = 16 > 0$$

$(0, \pm 1)$ är stabila noder.

$$\mathbf{g}(\pm 1, 0) = \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix} \quad \tau = 0$$

$(\pm 1, 0)$ är sadelpunkter.

Alternativ lösning.

$$\frac{dx}{dt} = x(1 - x^2 - 3y^2)$$

$$\frac{dy}{dt} = y(3 - x^2 - 3y^2)$$

Kritiska punkter:

$$\frac{dx}{dt} = x(1 - x^2 - 3y^2) = 0$$
$$\frac{dy}{dt} = y(3 - x^2 - 3y^2) = 0$$

$$(0,0), (0, \pm 1), (\pm 1, 0)$$

$(0,0)$

$$\begin{array}{l} x \\ y \end{array} = \begin{array}{cc} 1 & 0 \\ 0 & 3 \end{array} \begin{array}{l} x \\ y \end{array} + \begin{array}{l} x(-x^2 - 3y^2) \\ y(-x^2 - 3y^2) \end{array}$$

$$\mathbf{A}_1 = \begin{array}{cc} 1 & 0 \\ 0 & 3 \end{array}, \det(\mathbf{A}_1 - \lambda \mathbf{I}) = \mathbf{0} \quad \begin{array}{l} \lambda_1 = 1 \\ \lambda_2 = 3 \end{array}$$

$$K_1 = \begin{array}{c} 1 \\ 0 \end{array}, K_2 = \begin{array}{c} 0 \\ 1 \end{array}$$

Instabil nod.

$(0,1)$

Sätt:
$$\begin{array}{ll} u = x - 0 & u = x \\ v = y - 1 & v = y \end{array} \cdot$$

$$\frac{u}{v} = \frac{u(1 - u^2 - 3(v + 1)^2)}{(v + 1)(3 - u^2 - 3(v + 1)^2)}$$

$$\frac{u}{v} = \frac{-2u}{-6v} + \frac{-u^3 - 6uv - 3uv^2}{-u^2v - u^2 - 3v^3 - 3v^2 - 6v^2}$$

$$\begin{matrix} u \\ v \end{matrix} = \begin{matrix} -2 & 0 \\ 0 & -6 \end{matrix} \begin{matrix} u \\ v \end{matrix} + \begin{matrix} O(\rho^2) \\ O(\rho^2) \end{matrix}$$

$$\mathbf{A}_2 = \begin{matrix} -2 & 0 \\ 0 & -6 \end{matrix}, \det(\mathbf{A}_2 - \lambda \mathbf{I}) = \mathbf{0} \quad \begin{matrix} \lambda_1 = -2 \\ \lambda_2 = -6 \end{matrix}$$

$$K_1 = \begin{matrix} 1 \\ 0 \end{matrix}, K_2 = \begin{matrix} 0 \\ 1 \end{matrix}$$

Stabil nod.

$(0, -1)$

Sätt:

$$\begin{array}{ll} u = x - 0 & u = x \\ v = y + 1 & v = y \end{array} \cdot$$

$$\begin{array}{l} u \\ v \end{array} = \begin{array}{l} u(1 - u^2 - 3(v - 1)^2) \\ (v - 1)(3 - u^2 - 3(v - 1)^2) \end{array}$$

$$\begin{array}{l} u \\ v \end{array} = \begin{array}{l} -2u \\ -6v \end{array} + \begin{array}{l} O(\rho^2) \\ O(\rho^2) \end{array}$$

$$\begin{matrix} u \\ v \end{matrix} = \begin{matrix} -2 & 0 \\ 0 & -6 \end{matrix} \begin{matrix} u \\ v \end{matrix} + \begin{matrix} O(\rho^2) \\ O(\rho^2) \end{matrix}$$

$$\mathbf{A}_2 = \begin{matrix} -2 & 0 \\ 0 & -6 \end{matrix}, \quad \det(\mathbf{A}_2 - \lambda \mathbf{I}) = \mathbf{0} \quad \begin{matrix} \lambda_1 = -2 \\ \lambda_2 = -6 \end{matrix}$$

$$K_1 = \begin{matrix} 1 \\ 0 \end{matrix}, \quad K_2 = \begin{matrix} 0 \\ 1 \end{matrix}$$

Stabil nod.

(1,0)

Sätt:
$$\begin{array}{ll} u = x - 1 & u = x \\ v = y - 0 & v = y \end{array} \cdot$$

$$\begin{array}{l} u \\ v \end{array} = \begin{array}{l} (1 + u)(1 - (1 + u)^2 - 3v^2) \\ v(3 - (1 + u)^2 - 3v^2) \end{array}$$

$$\begin{array}{l} u \\ v \end{array} = \begin{array}{l} -2u \\ 2v \end{array} + \begin{array}{l} O(\rho^2) \\ O(\rho^2) \end{array}$$

$$\begin{matrix} u \\ v \end{matrix} = \begin{matrix} -2 & 0 \\ 0 & 2 \end{matrix} \begin{matrix} u \\ v \end{matrix} + \begin{matrix} O(\rho^2) \\ O(\rho^2) \end{matrix}$$

$$\mathbf{A}_3 = \begin{matrix} -2 & 0 \\ 0 & 2 \end{matrix}, \quad \det(\mathbf{A}_3 - \lambda \mathbf{I}) = 0 \quad \begin{matrix} \lambda_1 = -2 \\ \lambda_2 = 2 \end{matrix}$$

$$K_1 = \begin{matrix} 1 \\ 0 \end{matrix}, \quad K_2 = \begin{matrix} 0 \\ 1 \end{matrix}$$

Sadelpunkt

$(-1, 0)$

Sätt:
$$\begin{array}{ll} u = x + 1 & u = x \\ v = y - 0 & v = y \end{array} \cdot$$

$$\begin{array}{l} u \\ v \end{array} = \begin{array}{l} (-1 + u)(1 - (-1 + u)^2 - 3v^2) \\ v(3 - (-1 + u)^2 - 3v^2) \end{array}$$

$$\begin{array}{l} u \\ v \end{array} = \begin{array}{l} -2u \\ 2v \end{array} + \begin{array}{l} O(\rho^2) \\ O(\rho^2) \end{array}$$

$$\begin{array}{l} u \\ v \end{array} = \begin{array}{cc} -2 & 0 \\ 0 & 2 \end{array} \begin{array}{l} u \\ v \end{array} + \begin{array}{l} O(\rho^2) \\ O(\rho^2) \end{array}$$

$$\mathbf{A}_3 = \begin{array}{cc} -2 & 0 \\ 0 & 2 \end{array}, \det(\mathbf{A}_3 - \lambda \mathbf{I}) = \mathbf{0} \quad \begin{array}{l} \lambda_1 = -2 \\ \lambda_2 = 2 \end{array}$$

$$K_1 = \begin{array}{c} 1 \\ 0 \end{array}, K_2 = \begin{array}{c} 0 \\ 1 \end{array}$$

Sadelpunkt