12.3.4.

$$k\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}\ ,\ 0<x<L\ ,\ t>0\ .$$

$$\text{Randvillkor}:\begin{cases}\dfrac{\partial u}{\partial x}(0,t)=0\\[2mm] \dfrac{\partial u}{\partial x}(L,t)=0\end{cases}\qquad t>0\ .$$

$$\text{Begynnelsevillkor}: u(x,0)=f(x)=\begin{cases}x\ ,\ 0<x<1\\ 0\ ,\ 1<x<2\end{cases}\ ,\ 0<x<2\ .$$

$$u(x,t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{2}e^{-(\frac{n\pi}{2})^2 kt}$$

Begynnelsevillkoret ger:

$$f(x) = u(x,0) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{2}$$

$$a_0 = \frac{2}{2}\int_0^2 f(x)dx = \int_0^1 xdx = \frac{1}{2}$$

$$a_n = \frac{2}{2}\int_0^2 f(x)\cos\frac{n\pi x}{2}dx = \int_0^1 x\cos\frac{n\pi x}{2}dx = \{\text{part. int.}\} =$$

$$= \left[x \frac{2}{n\pi} \sin\frac{n\pi x}{2} \right]_0^1 - \int_0^1 1 \frac{2}{n\pi} \sin\frac{n\pi x}{2}\, dx =$$

$$= \frac{2}{n\pi} \sin\frac{n\pi}{2} + \left[\left(\frac{2}{n\pi} \right)^2 \cos\frac{n\pi x}{2} \right]_0^1 =$$

$$= 4\left\{ \frac{1}{2n\pi} \sin\frac{n\pi}{2} + \frac{1}{n^2\pi^2} \left(\cos\frac{n\pi}{2} - 1 \right) \right\}$$

$$u(x,t) = \frac{1}{4} + \sum_{n=1}^{\infty} 4\left\{ \frac{1}{2n\pi} \sin\frac{n\pi}{2} + \frac{1}{n^2\pi^2} \left(\cos\frac{n\pi}{2} - 1 \right) \right\} \cos\frac{n\pi x}{2}\, e^{-(\frac{n\pi}{2})^2 kt}$$