8.3.22.

$$\mathbf{X}' = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 1/t \\ 1/t \end{pmatrix}, \quad \mathbf{X}(1) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

Bestäm en fundamentalmatris, $\Phi(t)$.

$$\mathbf{X}' = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \mathbf{X}$$

$$0 = \begin{vmatrix} 1-\lambda & -1 \\ 1 & -1-\lambda \end{vmatrix} = \lambda^2$$

$$\lambda_{1,2} = 0$$

Insättning i $(\mathbf{A}-\lambda\mathbf{I})\mathbf{v}=\mathbf{0}$ ger:

$$\begin{pmatrix}1 & -1\\ 1 & -1\end{pmatrix}\mathbf{v}_1=\mathbf{0}$$

$$\mathbf{v}_1=\begin{pmatrix}1\\ 1\end{pmatrix}$$

$$\begin{pmatrix}1 & -1\\ 1 & -1\end{pmatrix}\mathbf{P}=\begin{pmatrix}1\\ 1\end{pmatrix}$$

$$\mathbf{P}=\begin{pmatrix}1\\ 0\end{pmatrix}$$

$$\mathbf{X}_1=\begin{pmatrix}1\\ 1\end{pmatrix},\quad \mathbf{X}_2=\begin{pmatrix}1\\ 1\end{pmatrix}t+\begin{pmatrix}1\\ 0\end{pmatrix}=\begin{pmatrix}t+1\\ t\end{pmatrix}$$

En fundamentalmatris $\Phi(t)=\begin{pmatrix}1 & t+1\\ 1 & t\end{pmatrix}$.

$$\mathbf{X}_{\mathbf{p}} = \Phi(t)\mathbf{U} = \Phi(t)\int \Phi^{-1}(t)\mathbf{F}(t)\mathbf{dt}$$

$$\Phi^{-1}(t) = \frac{1}{-1}\begin{pmatrix} t & -(t+1) \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} -t & t+1 \\ 1 & -1 \end{pmatrix}$$

$$\mathbf{U} = \int \begin{pmatrix} -t & t+1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1/t \\ 1/t \end{pmatrix}\mathbf{dt} = \int \begin{pmatrix} 1/t \\ 0 \end{pmatrix}\mathbf{dt} = \begin{pmatrix} \ln t \\ 0 \end{pmatrix}$$

$$\mathbf{X}_{\mathbf{p}} = \begin{pmatrix} 1 & t+1 \\ 1 & t \end{pmatrix}\begin{pmatrix} \ln t \\ 0 \end{pmatrix} = \begin{pmatrix} \ln t \\ \ln t \end{pmatrix}$$

$$\mathbf{X} = \Phi(t)\mathbf{C} + \Phi(t)\mathbf{U} = \begin{pmatrix} 1 & t+1 \\ 1 & t \end{pmatrix}\mathbf{C} + \begin{pmatrix} \ln t \\ \ln t \end{pmatrix}$$

$$\mathbf{X}(1) = \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \mathbf{C} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

$$\mathbf{X} = \begin{pmatrix} 1 & t+1 \\ 1 & t \end{pmatrix} \begin{pmatrix} -4 \\ 3 \end{pmatrix} + \begin{pmatrix} \ln t \\ \ln t \end{pmatrix} = t \begin{pmatrix} 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \ln t \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$