

Section number 12 — Probability and Statistics

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## STOCHASTIC PROPERTIES OF HÖLDER AND FRACTIONAL FIELDS

In the problems of stochastic analysis and in the investigation of some physical problems, in particular hydromechanics, it is necessary to construct a field which is two-parameter random functional with stationary increments. Fractional Brownian fields are examples of these functions. These processes model share price dynamics in exchange subject to same waves of randomness. If this randomness has long-term dependence, then it is best modelled by fractional Brownian field. This way processes are modelled when making decisions on financial investments, calculating insurance premiums, working with resources in industry. To consider, for example, stochastic differential equation with fractional Brownian field it is necessary at first to construct the theory of integration with respect to such fields.

In the present work, the definition of fractional two-parameter integrals and Weyl, Liouville, Marschaud fractional derivatives is given. The definition, main properties and some formulas for calculation of fractional generalized two-parameter Lebesgue-Stieltjes integrals are obtained.

Since fractional Brownian fields have Hölder properties of their trajectories, it is natural to establish the properties of generalized stochastic integrals with respect to Hölder fields that is done and applied to fractional fields. It is established that for such fields the generalized Lebesgue-Stieltjes integrals are in fact the limits of Riemann-Stieltjes sums. Then, Ito formula is obtained under rather weak conditions of the reference function. Some auxiliary inequalities are derived for Hölder norms of generalized stochastic integrals. Stochastic differential equations involving fractional Brownian fields are considered.

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