

ABSTRACT

Let X be a non-empty set and $f : X \rightarrow X$ a map. Considering f as $1 - ary$ operation on X , one can study the algebraic structure (X, f) , which is called “*monounary algebra*”.

Many authors study problems concerning monounary algebras but some properties are not clearly proved especially in the finite case.

Of course one can deal with this kind of structures as one generally deals with structures based on binary operations examining subalgebras, congruences and quotient algebras, simple algebras, ideals, homomorphisms and automorphisms.

But these algebras can be introduced as particular oriented graphs too, emphasizing their topological or combinatorial properties.

In doing so, one can show precise connections between these two kinds of approaches and pair off results and proof-techniques from one sphere to the other.

In the finite case one can obtain methods of algebraic representation (even if not faithful) through adjacency matrices or polynomials, which are sources of enumeration and classification problems.

We classify the so called “simple” monounary algebras and we characterize those algebras with specific subalgebra lattices and in particular those for which the $1 - ary$ operation is bijective.

Besides we illustrate the situation from the point of view of graphs. The connected components will be determined and it will be shown their $1 - 1$ correspondence with the minimal subalgebras.

At this point, the symbiosis between the two points of view will give the opportunity to classify the maximal subalgebras, their automorphism group etc.

Finally we study representations of algebras in terms either of matrices or of polynomials.

References

- Cohn P.M., Universal Algebra, D. Reidel Publishing Company 1981.
- Harary F., Graph Theory, Addison-Wesley Publishing Company 1972.
- Jakubínková-Studenovská, D.: Monounary algebras and bottleneck algebras *Algebra Universalis* 40,59-72 (1998).
- Jónsson, B.: Topics in Universal Algebra, Lecture Notes in Mathematics **250**, Springer-Verlag, Berlin (1972).
- R. Schmidt, Subgroup Lattice of Groups, de Gruyter 1994.
- W.R. Scott, Group Theory, Prentice-Hall 1964.
- W.Sierpinski: Elementary Theory of Numbers, Państwowe Wydawnictwo Naukowe, Warszawa 1964.
- G. Szasz, Introduction to Lattice Theory, Acad. Press 1963.