ON MINIMAL COMPONENTS OF EQUIDIMENSIONAL SYMMETRIC ALGEBRAS

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Abstract

This work is devoted to describing minimal components of equidimensional symmetric algebras. The proof combines results on prime submodules of modules and the identity that Matsumura calls the "dimension formula".

Let R be a universally catenarian Noetherian domain, let N be a finitely generated R-module such that the symmetric algebra $S(N)$ is equidimensional. If $\mathfrak{p}$ is a prime ideal of R we denote the least number of generators of $N_{\mathfrak{p}}$ by $\nu(N_{\mathfrak{p}})$ and let f denote the natural map from $SpecS(N)$ to $Spec(R)$.

Given $\mathfrak{p} \in Spec(R)$, the purpose of this paper is to prove that in $f^{-1}(\mathfrak{p})$ there is a minimal prime ideal of $S(N)$ if and only if $\nu(N_{\mathfrak{p}}) - ht\mathfrak{p} = rankN$. In case this condition fulfils for $\mathfrak{p}$, there exists only one a minimal prime ideal of $S(N)$ in $f^{-1}(\mathfrak{p})$ denoted by $\mathcal{E}_{\mathfrak{p}(0)}$ and defined as

$$\mathcal{E}_{\mathfrak{p}(0)} = \{b \in S(N) : ab \in \mathfrak{p} \cdot S(N) \text{ for some } a \in R - \mathfrak{p}\}$$