Systems of germs and theorems of zeros in infinite-dimensional spaces

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In this paper we study the infinite-dimensional analytic (complex and real) case, where an analytic function depends only on a finite number of variables (is *finitely presented*). But, as the number of all variables is infinite, the ring of the germs of such functions is no longer Noetherian and the germs of the zero sets of ideals cannot be defined in the standard way. Moreover, as we have shown in [5], there is no topology in the infinite-dimensional space of all complex or real sequences that would give required properties of the germs of sets. We have been interested there in "local" solutions of infinitely many analytic equations in infinitely many variables. Such equations describe, for instance, indistinguishable states of a (control) system with output and are related to *observability* and *local observability* of the system (see e.g. [1, 2] for the finite-dimensional case and [4, 5, 6] for the infinite-dimensional one).

Instead of using topology to define the germ of a set, we consider special families of finite dimensional set-germs (*systems of germs*) which approximate in some sense what we want to be an infinite-dimensional set-germ. Systems of germs give rise to a concept of multigerm — the equivalence class of such systems under a natural equivalence relation.

The main result of this paper consists of real and complex theorems of zeros, where we show that the real or ordinary radical of an ideal consists exactly of the germs of finitely presented analytic functions (real or complex, respectively) that vanish on the multigerm of zeros (real or complex again) of the ideal.

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