Homology structure of ergodic measures

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Any Lagrangian function on a closed finite-dimensional manifold M, when depending 2π- periodically on the evolution parameter generates so called Lagrangian flow. Its related group of diffeomorphisms na $T(M) \times \mathbb{S}^1$ makes it possible to constuct the set of normed (probabilistic) invariant measures on $T(M) \times \mathbb{S}^1$. The latter appears to be a convex set completely characterized by means of so called extreme points being at the same time due to a result of J. Mather ergodic measures of the Lagrangian flow under regard. On the other hand, there exists a natural mapping from the space of all invariant measure space mentioned above into the first homology group $H(M;\mathbb{R})$ of the manifold M via a well known Mather's construction, whose image is exactly the measure homology of our Lagrangian system. Its properties appeared to be very very important for detecting the corresponding ergodic mesures, making use a new tool of its studing related with so called Legendrian transformations and Poincare -Cartan invariants. Moreover in the case when our Lagrangian funcion depends adiabatically on a small parameter $\varepsilon \downarrow 0$ through the expession $\varepsilon t \in \mathbb{R}/2\pi\mathbb{Z}$, a suitable application of a bit generalized Legendrian transformation together with the technique of Poincare -Cartan invariants makes it possible to investiagte the existence and properties of so called adiabatic invariants and ergodic measures on $T(M) \times \mathbb{S}^1$. There is developed an approach to studying ergodic properties of time-dependent periodic Hamiltonian flows on symplectic metric manifolds having applications in mechanics and mathematical physics. Based both on J. Mather's results about homology of probability invariant measures minimizing some Lagrangian functionals and on the symplectic field theory devised by A. Floer and others for investigating symplectic actions and Lagrangian submanifold intersections, an analog of Mather's β-function is constructed subject to a Hamiltonian flow reduced invariantly upon some compact neighborhood of a Lagrangian submanifold. Some of results, being done for stable and unstable manifolds to hyperbolic orbits and having applications in the theory of adiabatic invariants of slowly perturbed integrable Hamiltonian systems, are stated within the Gromov-Salamon-Zehnder elliptic techniques in symplectic geometry. These properties are studied simultaneously making use also of the theory of spectral invariants applied to the generator of the corresponding Hamiltonian flow on the symplectic phase space $T^*(M)$.

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