Two-dimensional polynomial dynamical systems are considered. The main problem of the qualitative theory of such systems is Hilbert's sixteenth problem on the maximum number and relative position of limit cycles. There are three local bifurcations of limit cycles: 1) Andronov – Hopf bifurcation (from a singular point of center or focus type); 2) separatrix cycle bifurcation (from a homoclinic or heteroclinic orbit); 3) multiple limit cycle bifurcation (from a multiple limit cycle of even or odd multiplicity). We connect all these local bifurcations by means of the Wintner – Perko termination principle and develop a new global approach to the solution of Hilbert's sixteenth problem. Earlier this approach was applied to the global qualitative analysis of quadratic dynamical systems [1]. Now it is used for the study of cubic systems.

In particular, using Erugin's two-isocline method [1], we construct a canonical cubic system of Kukles type and carry out the global analysis of its special case corresponding to a generalized Liénard equation. We prove that the foci of such a Liénard system can be at most of second order, and that this particular system can have at least three limit cycles in the whole phase plane. Moreover, unlike all previous works on the Kukles-type systems, we study global bifurcations of limit and separatrix cycles, using arbitrary (including as large as possible) field-rotation parameters of our canonical system. As a result, we have obtained a classification of all possible types of separatrix cycles for this generalized Liénard system and also all possible distributions of its limit cycles, conjecturing that this system has at most three limit cycles.

References

[1] V. A. Gaiko, *Global Bifurcation Theory and Hilbert's Sixteenth Problem*, Kluwer, Boston, 2003.