The periodic problem for a Hamiltonian system of the form

$$(A+\varepsilon B)\frac{dx}{dt}=\left(\frac{\partial H}{\partial y}\right)', (A'+\varepsilon B')\frac{dy}{dt}=-\left(\frac{\partial H}{\partial x}\right)', \tag{1}$$

where the prime denotes the transposition, appears from the control optimality condition for a periodic problem of minimizing of the functional on trajectories of the equation with the operator $A+\varepsilon B$ standing before the derivative. Here A is singular and $A+\varepsilon B$ is invertible for sufficiently small ε. Under certain conditions, the asymptotic solution of the periodic problem for the system (1) is constructed in the form of the series with respect to non-negative integer powers of ε.

In the paper [1], the basic assumption for the investigation of singularly perturbed $T-$ periodic problems is the reducibility of the matrix, standing before the unknown in the linearized equation for the fast variable, to a block-diagonal form by a real smooth matrix of the same period T and, in the resulting block-diagonal form, the spectrum of one of the matrices on the diagonal belongs to the open left half-plane and that of the other to the open right half-plane. The example, showing that such reducibility is not possible always, is given in [2]. It appears that the condition of the reducibility for a Hamiltonian operators is correctly provided the spectra of some operators do not intersect the imaginary axis (see [3]).

References

[1] Flatto L., Levinson N. J. Rational Mech. Analysis. 1955. Vol.4. P.943-950.
[2] Sibuya Y. Math. Ann. 1965. Vol.161. P.67-77.
[3] Kurina G.A., Martynenko G.V. Matematicheskie Zametki. 2003. Vol.74. No.5. P.789-792 (in Russian).