Section number: 08

Peter Zhidkov
Bogoliubov Laboratory of Theoretical Physics, Joint Institute
for Nuclear Research, 141980 Dubna (Moscow region), Russia

Email: zhidkov@thsun1.jinr.ru

BASIS PROPERTIES OF EIGENFUNCTIONS OF NONLINEAR STURM-LIOUVILLE PROBLEMS

We consider a number of nonlinear Sturm-Liouville-type problems and establish results on basis properties in L_2 for systems of their eigenfunctions (or solutions). The following problem is one of those we deal with:

$$-u'' + f(u^2)u = \lambda u, \quad u = u(x), \quad x \in (0,1), \quad u(0) = u(1) = 0, \quad u'(0) > 0, \quad \int_0^1 u^2(x)dx = 1.$$

Here all quantities are real, λ is a spectral parameter, and f is a given function. The result for this problem is that if $f(s)$ is a smooth nondecreasing function of $s \geq 0$, then for any nonnegative integer n there exists a unique eigenfunction u_n that has precisely n zeros in the interval $(0,1)$ and the sequence $\{u_n\}_{n=0,1,2,...}$ of all eigenfunctions is a basis (in addition, a Bari basis) in $L_2(0,1)$. Also, we consider an analog of the Fourier transform associated with a nonlinear problem on a half-line.

AMS subject classification numbers (2000): 34L10, 34L30, 34L99

Keywords: nonlinear Sturm-Liouville-type problem, eigenfunction expansion, Riesz basis, Bari theorem, Fourier transform