Compressible dusty gas model: global solvability and stability

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There are a number of mathematical models describing two-phase flows of gas-particles mixtures, known as "dusty gases". Within various physical and numerical studies, the qualitative mathematical analysis (i.e., well-posedness, uniqueness, global solvability, stability, etc.) of such models is not satisfactory. The present communication deals with a dusty gas model in which the carrier phase is assumed to be a compressible viscous fluid with the velocity u and density ρ, and the dust is described by hyperbolic conservation laws for velocity v and local concentration m of the dust particles. The pressure p of a clean gas satisfies the polytropic equation of state: $p(\rho) = \rho^\gamma$ with $\gamma > 1$. The particles are assumed of uniform size, shape and mass.

To simplify the model we suppose that the dust concentration is so small that the net effect of the dust on the gas is equivalent to a force $K(v-u)\int m(x,t)\,dx$, where K is a positive constant, and $\int m(x,t)\,dx$ means the averaged concentration of the dust. It should be noted that, in general, this force is written either as $mK(v-u)$, or simply as $K(v-u)$, and both these cases are discussible from the physical point of view.

Under these assumptions, the equations of motion for the model are:

$$\rho_t + \nabla\cdot(\rho u) = 0,$$
$$\rho\Big(u_t + (u\cdot\nabla)u\Big) + \nabla p = \mu\Delta u + (\lambda+\mu)\nabla(\nabla\cdot u) + K(v-u)\int_\Omega m(x,t)\,dx,$$
$$v_t + (v\cdot\nabla)v = K(u-v),$$
$$m_t + \nabla\cdot(mv) = 0,$$

where λ and μ are the viscosity coefficients of a clean gas and $\Omega\subset\mathbb{R}^n$ is a bounded domain of the flow.

We prove global-in-time unique solvability and exponential decay of the energy for this model in one-dimensional case provided sufficiently small initial data. These results can be interpreted as an asymptotic stability of a steady state of a dusty gas flow.