SINGULAR APPROXIMATIONS TO HYPERBOLIC SYSTEMS OF CONSERVATION LAWS IN ONE SPACE DIMENSION

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Consider a $n \times n$ hyperbolic system of conservation laws of the form

$$u_t + f(u)_x = 0, \qquad (t,x) \in \mathbb{R}^+ \times \mathbb{R}, \quad u \in \mathbb{R}^n, \tag{1}$$

Here $u = (u_1, \dots, u_n)$ is the vector of *conserved quantities*, while the components of $f = (f_1, \dots, f_n)$ are the *fluxes*. The system is said strictly hyperbolic if at each point u the Jacobian matrix $Df(u)$ has n real, distinct eigenvalues

$$\lambda_1(u) < \cdots < \lambda_n(u).$$

Several fundamental laws of physics take the form of a conservation equation.

Weak solutions to the Cauchy problem

$$u(0,x) = u_0(x) \tag{2}$$

were constructed using *ad hoc* method, mainly the Glimm scheme and the front tracking approximations. These schemes require some additional technical assumptions on the flux function f.

Aim of this talk is to present a different approach to the construction of solutions to (1), (2). We will consider in fact system (1) as the limit as $\epsilon \to 0$ of one of the following systems:

(1) $n \times n$ parabolic system of the form

$$u_t + f(u)_x = \epsilon u_{xx};$$

(2) semidiscrete schemes, for example the upwind scheme

$$u_t(t,x) + \frac{1}{\epsilon}\big(f(u(t,x)) - f(u(t,x-\epsilon))\big) = 0,$$

or the backward scheme

$$\frac{1}{\epsilon}\big(u(t,x) - u(t-\epsilon,x)\big) + f(u(t,x)) = 0;$$

(3) relaxation approximation, in particular

$$\begin{cases} u_t + v_x &= 0 \\ v_t + \Lambda 2 u_x &= (f(u) - v)/\epsilon. \end{cases}$$

All these approximations are interesting from the physical and numerical point of view.

We will show that as $\epsilon \to 0$ the solution of all different schemes converges to a unique weak solution to (1) (independently on the scheme considered). This solution is in the class of function where one can prove well posedness in L^1 of the Cauchy problem (1), (2), showing that this class contains the ”good” solution, i.e. solutions which satisfy entropy principles. The results we obtain do not require any additional assumption on the flux f besides strict hyperbolicity.